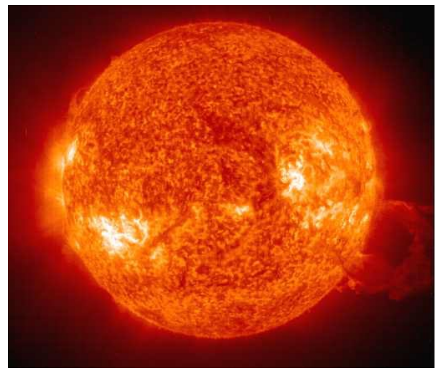
Thermal emission of EM radiation



'The birth of photons'

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Lecture overview

- Shortwave (solar) radiation
  - Principle source is the sun
  - Distribution governed by *absorption, transmission and reflection*
- Longwave radiation
  - Principle source is *thermal emission* by the Earth and atmosphere
  - Conversion of internal energy to radiant energy
- Blackbody radiation
- Radiation Laws
- Emissivity

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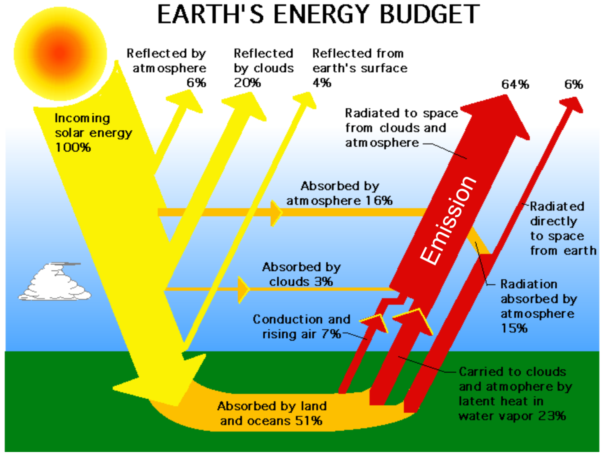
Earth's energy budget



- Earth's radiation budget is the balance between incoming solar radiation and outgoing (reflected and emitted) energy from the Earth
- Governs Earth's surface temperature and the greenhouse effect

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Kirchhoff's Law

$$\varepsilon_\lambda(\theta,\phi) = a_\lambda(\theta,\phi)$$

- Proposed by Gustav Kirchhoff in 1859
- At thermal equilibrium, the emissivity of a body equals its absorptivity
- 'A good absorber is a good emitter'
- 'A good reflector is a poor emitter' (and vice versa), e.g., an emergency thermal blanket
- If a material is capable of absorbing a particular frequency, it follows that it will be a good emitter at the same frequency
- NB. *Nearly always an approximation* for real surfaces – absorptivity may depend on both wavelength and direction

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Blackbody radiation

• All objects above absolute zero (0 K or -273°C) radiate EM energy (due to vibration of atoms)

• EM emission dependent on temperature (atomic/molecular vibrations)

• What is the *theoretical maximum amount of radiation that can be emitted by an object*?

• **Blackbody** : a (hypothetical) body that absorbs all EM radiation incident on it

• Absorptivity (a) = 1 (and hence ε = 1, from Kirchhoff's Law)

• The emissivity of a material is the ratio of what it emits to what *would be* emitted if it were a blackbody (ε = 0 for a *whitebody*, ε = 1 for a *blackbody*, 0 < ε < 1 for a *graybody*)

• Blackbodies are used to calibrate remote sensing instruments

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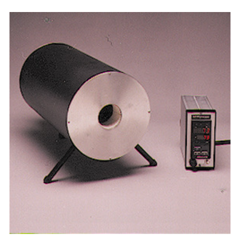
Blackbody radiation

• Blackbody radiation is isotropic and anything but black…

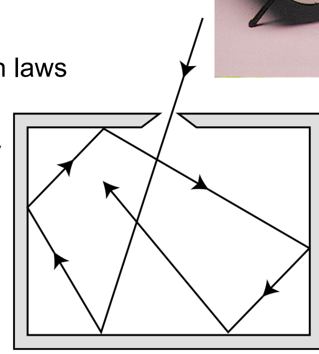
• The name comes from the assumption that the body absorbs at every frequency and hence would look black (but snow is almost a blackbody in the IR)



• Also called 'equilibrium radiation', 'cavity radiation' or 'radiation in equilibrium with matter'

• Foundation for development of radiation laws

Photons enter a cavity through a small hole. With each additional reflection from the cavity wall, the chance of absorption increases. If the hole is small and the cavity large, the absorptivity of the hole tends towards unity. The result is blackbody radiation at the temperature of the walls of the cavity.



What is the intensity of radiation emitted by a blackbody?

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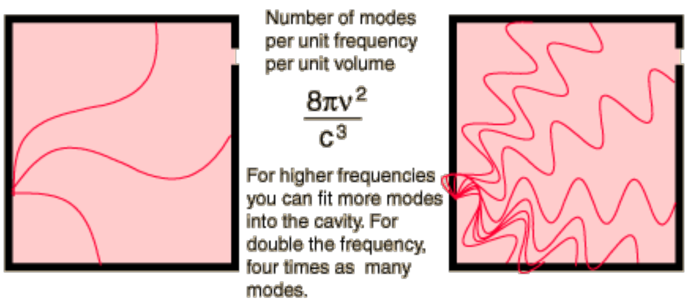
EM wave modes in a cavity



- The radiated energy from a cavity can be considered to be produced by standing waves or resonant modes of the cavity
- You can fit more modes of shorter wavelength (higher frequency) into a cavity
- Work by Lord Rayleigh and Sir James Jeans (in 1900-1905) showed that the number of modes was proportional to the frequency squared

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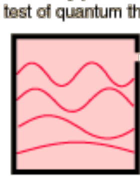
Blackbody radiation

Radiation modes in a hot cavity provide a test of quantum theory



	#Modes per unit frequency per unit volume	Probability of occupying modes	Average energy per mode
CLASSICAL	$\frac{8\pi\nu^2}{c^3}$	Equal for all modes	kT
QUANTUM	$\frac{8\pi\nu^2}{c^3}$	Quantized modes: require $h\nu$ energy to excite upper modes, less probable	$\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

- The amount of radiation emitted in a given frequency range should be proportional to the number of modes in that range
- ‘Classical’ (pre-quantum) physics suggested that all modes had an equal chance of being produced, and that the number of modes went up as the square of the frequency

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Rayleigh-Jeans Law

$$B_\lambda(T) = \frac{2ck_B}{\lambda^4} T$$

• Classical prediction of radiance from a blackbody at temperature T

• c: speed of light (2.998×10^{8} m s^{-1})

• k_B: Boltzmann's constant (1.381×10^{-23} J K^{-1})

• Blackbody emission proportional to temperature, and inversely proportional to the fourth power of wavelength

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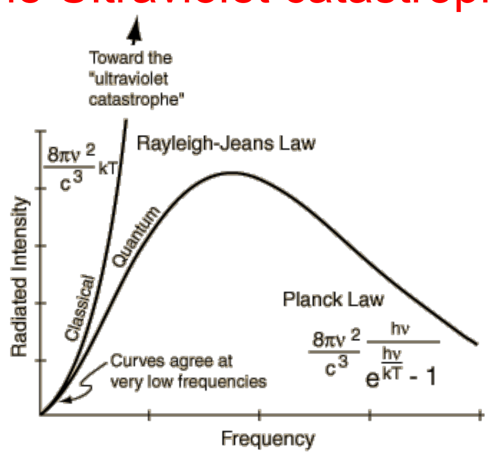
The Ultraviolet catastrophe



• The Rayleigh-Jeans law predicted a continued increase of radiated energy with increasing frequency and an 'ultraviolet catastrophe'

• Max Planck solved this (in 1901) by assuming that the EM modes in a cavity were quantized with energy (E) equal to hν (h = Planck's constant)

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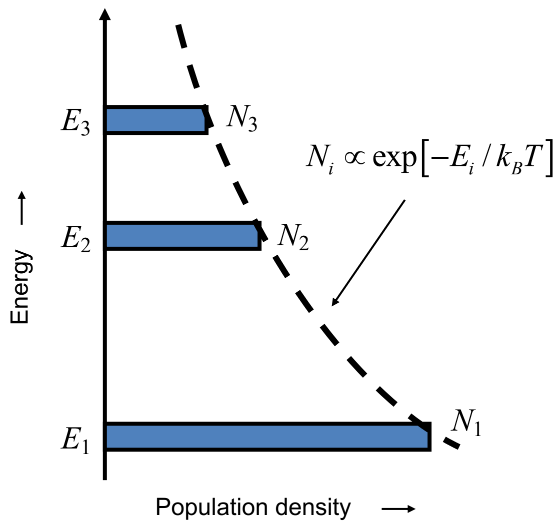
In what energy levels do molecules reside?

Boltzmann distribution



N_i is the number density of molecules in state i (i.e., the number of molecules per cm^3).

T is the temperature, and k_B is Boltzmann's constant - relates energy at the particle level to temperature.

Boltzmann's factor: exp[-E/kT] – expresses the probability of a state of energy E relative to a state of zero energy.

The probability of a given energy state decreases exponentially with energy.

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Planck's Function

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 \left(e^{hc/k_B \lambda T} - 1\right)}$$

- Radiation formula derived by Max Planck in 1901
- Gives the spectral radiance emitted by a blackbody at temperature T (K) *in the direction normal to the surface*
- Units?
- *c*: speed of light (2.998×10^{8} m s^{-1})
- *h*: Planck's constant (6.626×10^{-34} J s)
- k_B: Boltzmann's constant (1.381×10^{-23} J K^{-1}) – relates energy at the particle level to temperature
- Note that this form of the Planck function uses wavelength (λ)

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Planck's Function

$$B_\lambda(T) = \frac{c_1}{\lambda^5 (e^{c_2/\lambda T} - 1)}$$

- Collect constants together
- $c_1 = 1.19\times10^8$ W m^{-2} μm^4; $c_2 = 1.44\times10^4$ μm K
- Temperature in Kelvin (K), wavelength in microns for these constants
- **Need to be careful with units!**

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Planck's Function

$$B_\nu(T) = \frac{2h\nu^3}{c^2 (e^{h\nu/k_B T} - 1)}$$

- Planck's law as a function of frequency (v)
- Units: W m^{-2} Hz^{-1} sr^{-1}
- *c*: speed of light (2.998×10^8 m s^{-1})
- *h*: Planck's constant (6.626×10^{-34} J s)
- k_B: Boltzmann's constant (1.381×10^{-23} J K^{-1}) – relates energy at the particle level to temperature

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Spectral radiant exitance

$$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/k_B \lambda T} - 1)}$$

- Another form of the Planck function includes a factor of π
- This is emitted power integrated over all solid angles of a hemisphere (2π steradians) – *spectral radiant exitance*
- This is equivalent to multiplication by a factor of π (F = πI)

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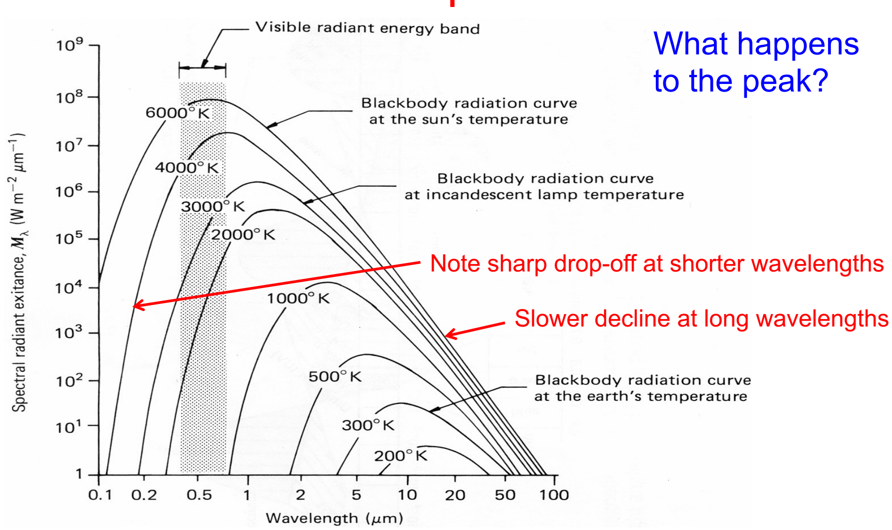
Planck curves for blackbodies at various temperatures



What happens to the peak?

http://webphysics.davidson.edu/alumni/MiLee/java/bb_mjl.htm - Blackbody applet

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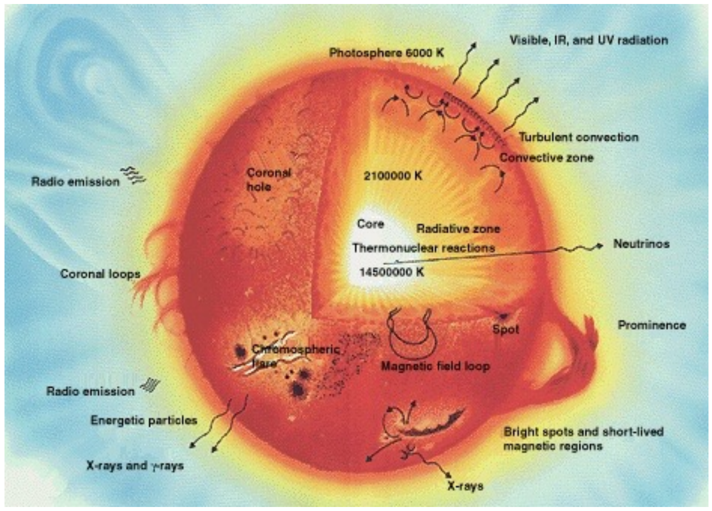
The Sun



- Fusion reactions in the Sun's core release high-energy photons (gamma rays)
- 'Random walk' of photons results in long photon travel time to the Sun's surface

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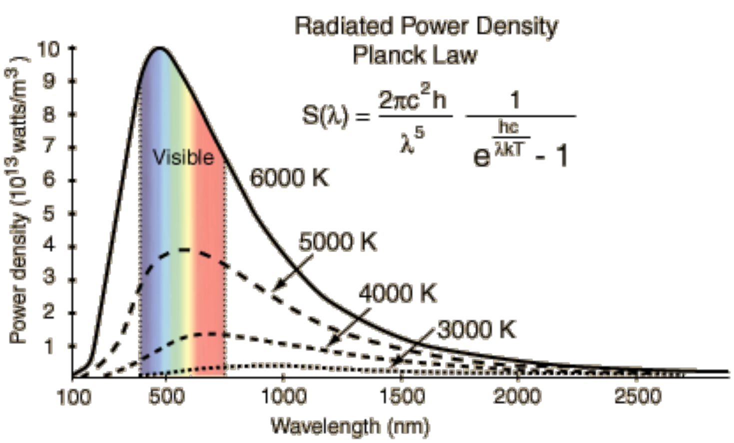
Planck curves – 6000 K



The Sun approximates a blackbody at a temperature of ~6000K
What color would the Sun be if viewed from outside the Earth's atmosphere?

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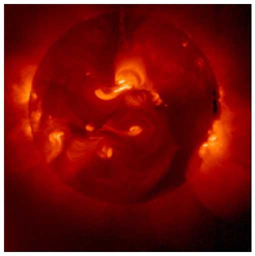
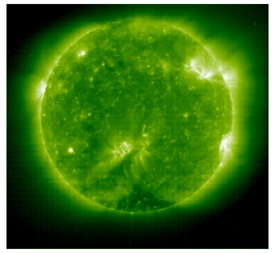
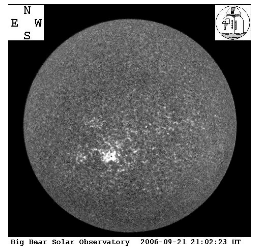
The Sun at different wavelengths



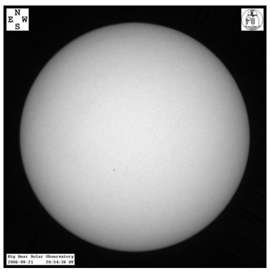
X-rays (0.3-4.5 nm)



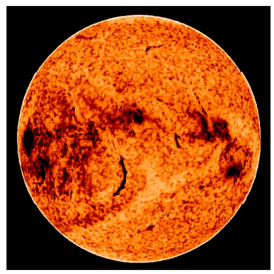
Extreme UV (19.5 nm)



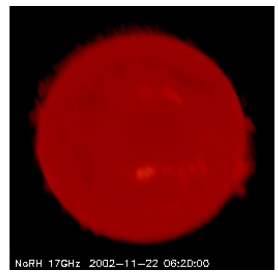
Blue light (393 nm)



Visible (‘white’) light



IR (1083 nm)



Microwave (1.7 cm)

http://www.helioviewer.org/

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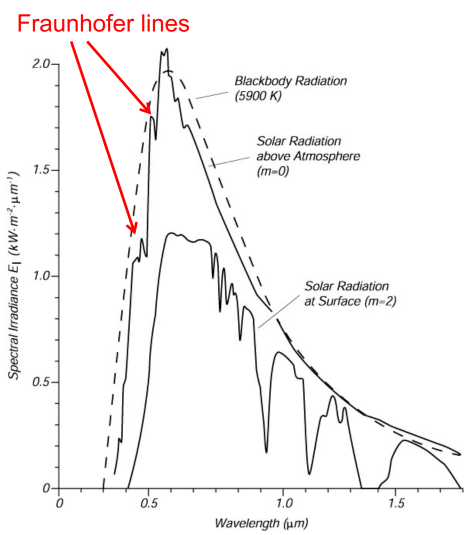
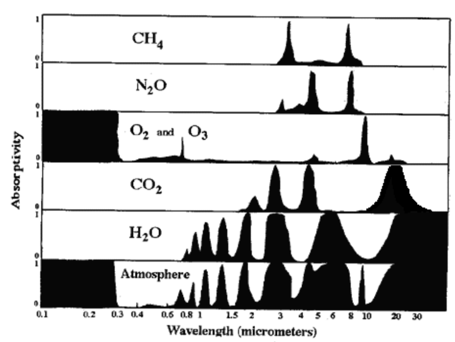
Departure from the blackbody assumption





Absorptivity of various gases of the atmosphere and the atmosphere as a whole as a function of the wavelength of radiation. An absorptivity of zero means no absorption while a value of one means complete absorption. The dominant absorbers of infrared radiation are water vapor (H_2O) and carbon dioxide (CO_2). Oxygen (O_2) and ozone (O_3) absorb much of the sun's ultraviolet radiation.

- Solar irradiance at the top of the atmosphere approximates a blackbody
- Lower in the atmosphere, attenuation by gases is important

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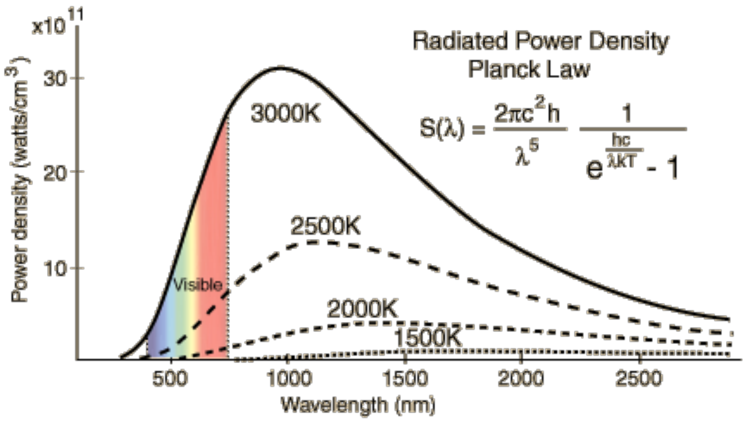
Planck curves – light bulb



Where is the peak of emission?

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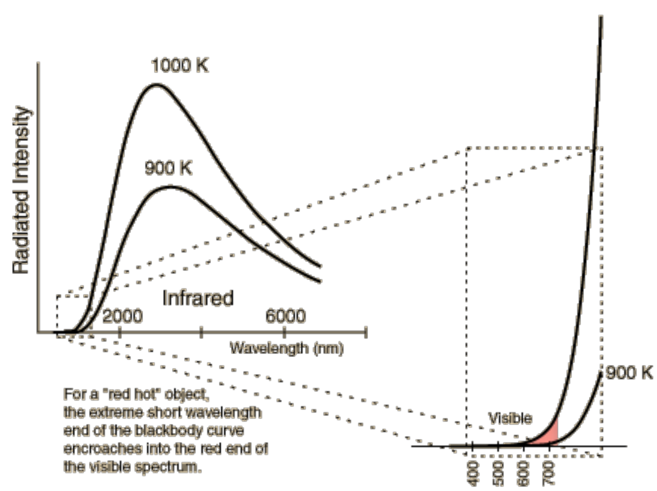
Planck curves – 'red hot'



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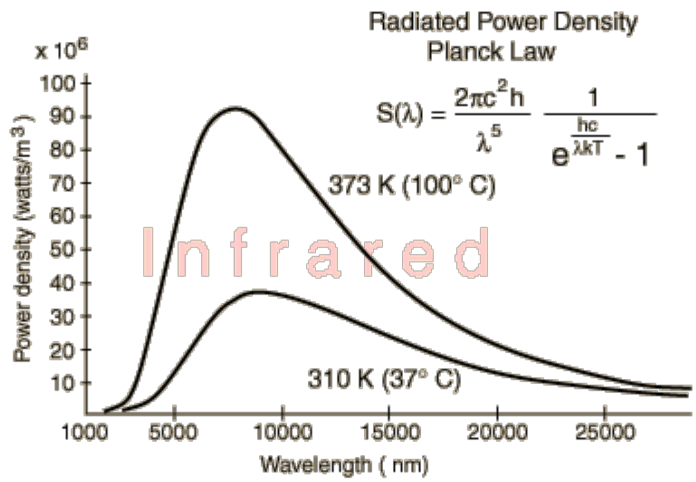
Planck curves – humans, terrestrial materials



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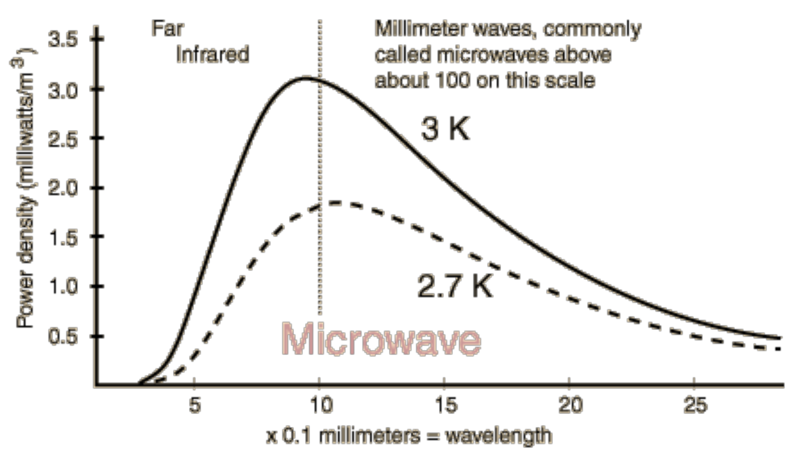
Planck curves – cosmic background



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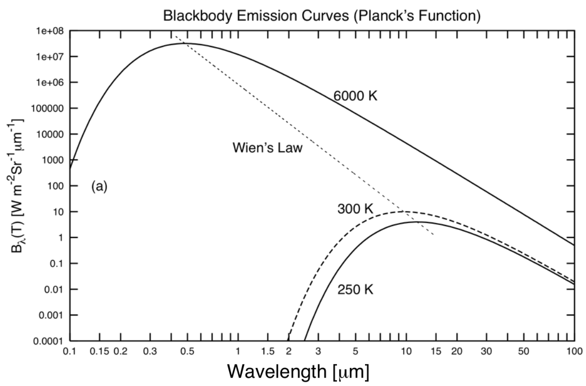
Wien's Law



$$\lambda_{\max} = \frac{k_W}{T}$$

- Also called Wien's Displacement Law: displacement of Planck curves
- Constant k_W = 2897 μm K (*for wavelength in μm*)
- Gives the wavelength λ_{max} of the peak of the Planck function for a blackbody at temperature T (in *Kelvin*)
- As temperature increases, the peak of the Planck function moves towards shorter wavelengths

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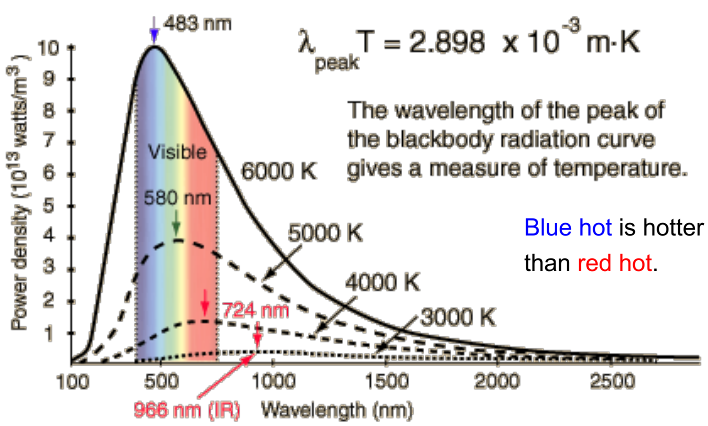
Wien's Law



Blue hot is hotter than red hot.

- Can be used to determine the temperatures of hot radiant objects from their radiation curves
- How would you derive Wien's Law from Planck's Function?

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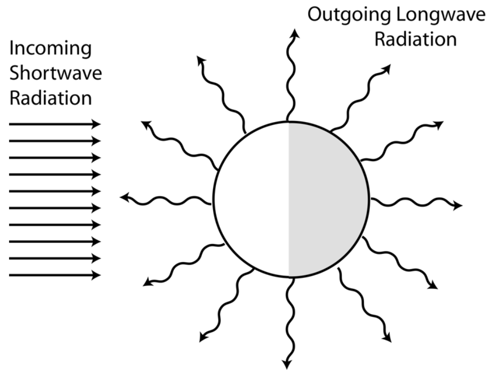
Broadband fluxes

• Planck's function gives the *monochromatic intensity* emitted by a blackbody

• For energy transfer in the Earth-atmosphere system, *broadband fluxes* are of more interest



• How do we obtain a broadband flux from Planck's function?

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Stefan-Boltzmann Law

$$F_{BB}(T) = \pi \int_0^\infty B_\lambda(T)\, d\lambda = \sigma T^4$$

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} \approx 5.67 \times 10^{-8} \frac{W}{m^2K^4}$$

• The *total broadband flux (or radiant exitance) from a blackbody at temperature T* (in W m^{-2})

• σ is called the Stefan-Boltzmann constant

• Derived by integrating the Planck function over all wavelengths and over the 2π steradians of solid angle of one hemisphere

• Note dependence on 4th power of temperature

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Stefan-Boltzmann Law derivation

$$F_{BB}(T) = \pi \int_0^\infty B_\lambda(T)\,d\lambda = \pi \int_0^\infty B_\nu(T)\,d\nu$$

or $$F_{BB}(T) = \int_0^\infty \frac{2\pi h\nu^3}{c^2(e^{h\nu/k_BT} - 1)} d\nu$$

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Stefan-Boltzmann Law derivation

- Make substitution: $x = \frac{h\nu}{k_BT}$ $\quad \nu = \frac{k_BTx}{h}$ $\quad d\nu = \frac{k_BT}{h}dx$

  (limits don't change)

Giving: $$F_{BB}(T) = \int_0^\infty \frac{2\pi h k_B^3 T^3 x^3}{c^2 h^3 (e^x - 1)} \frac{k_B T}{h} dx$$

Tidy up: $$F_{BB}(T) = \int_0^\infty \frac{2\pi k_B^4 T^4 x^3}{c^2 h^3 (e^x - 1)} dx$$

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Stefan-Boltzmann Law derivation

• Now removing constants from the integral gives:

$$F_{BB}(T) = \frac{2\pi k_B^4 T^4}{c^2 h^3} \int_0^\infty \frac{x^3 dx}{(e^x - 1)}$$

Evaluating the integral gives: $$\int_0^\infty \frac{x^3 dx}{(e^x - 1)} = \frac{\pi^4}{15}$$

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Stefan-Boltzmann Law derivation

• This leaves us with the Stefan-Boltzmann law:

$$F_{BB}(T) = \frac{2\pi^5 k_B^4 T^4}{15 c^2 h^3} = \sigma T^4$$

Where the Stefan-Boltzmann constant (σ) is ~5.67×10^{-8} W m^{-2} K^{-4}

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In-band radiance

- The Stefan-Boltzmann law gives us the total broadband flux from a blackbody
- What about the *flux between arbitrary wavelengths, e.g., $F_{\lambda 1 \rightarrow \lambda 2}$?*
- This requires integration of the Planck function between finite wavelength (or frequency, or wavenumber) limits
- This can be done:
  - Analytically (*hard*)
  - Using a table of integral values of the Planck function (*easier*)
  - Using an online spectral calculator (*easiest*)

http://www.spectralcalc.com/blackbody_calculator/blackbody.php

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Planck's function at longer wavelengths

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 (e^{hc/k_B \lambda T} - 1)}$$

- In the limit of longer λ, the term in the exponential becomes small
- Using the Taylor Series expansion: $e^x \approx 1 + x + x^2/2! + x^3/3! + x^4/4! + \ldots$
- For small x, this reduces to: $e^x \approx 1 + x$
- So we have:

$$B_\lambda(T) \approx \frac{2hc^2}{\lambda^5 (1 + \frac{hc}{k_B \lambda T} - 1)} \approx \frac{2ck_B T}{\lambda^4}$$

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Rayleigh-Jeans Approximation

$$B_\lambda(T) \approx \frac{2ck_B}{\lambda^4}T$$

- In the limit of large wavelength, the Rayleigh-Jeans approximation applies
- *c*: speed of light (2.998×10^8 m s^{-1})
- k_B: Boltzmann's constant (1.381×10^{-23} J K^{-1})
- *Blackbody emission directly proportional to temperature at longer λ*
- Valid for wavelengths of ~1 mm or longer (i.e., microwave remote sensing)
- Recall that this was the classical (pre-Planck) prediction of radiance from a blackbody at temperature T

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Emissivity (monochromatic)

$$\varepsilon_\lambda = \frac{I_\lambda}{B_\lambda(T)}$$

- Monochromatic emissivity (**ε_λ**) is the ratio of the observed monochromatic radiance at temperature T (***I_λ***) and wavelength **λ** to the (blackbody) radiance predicted by the Planck function (***$B_\lambda(T)$***)
- **ε_λ = 1** means that the surface is effectively a blackbody at that wavelength

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Emissivity (graybody)

$$\varepsilon = \frac{F}{\sigma T^4}$$

• Graybody emissivity (**ε**) is the ratio of the broadband flux (*F*) emitted by a surface to that predicted by the Stefan-Boltzmann Law.

• Note that no surface is truly 'gray' over the entire EM spectrum

Some typical infrared emissivities (in %)

Which best approximate blackbodies in the IR?

Water	92-96	Desert	90-91
Fresh, dry snow	82-99.5	Forest and shrubs	90
Ice	96	Human skin	95
Dry sand	84-90	Concrete	71-88
Moist sand	95-98	Polished aluminum	1-5
Dry, plowed soil	90		

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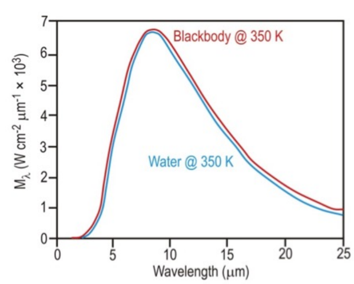
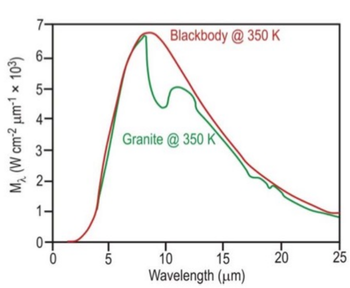
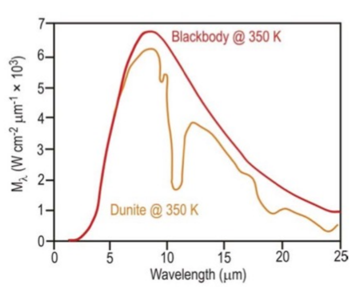
Emissivity of real materials







USGS Digital Spectral Library: http://speclab.cr.usgs.gov/spectral.lib06/ds231/datatable.html

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Inverting the Planck function

$$T_B = B_\lambda^{-1}(I_\lambda) \qquad T_B = \frac{c_2}{\lambda \ln\left(\dfrac{c_1}{\lambda^5 I_\lambda} + 1\right)}$$

T_B = Brightness Temperature

• If we know or measure the monochromatic radiance emitted by a material, 'inverting' the Planck function gives us the *equivalent blackbody temperature* (T_B) as a function of radiance (I_λ)

• $c_1 = 1.19\times10^{-16}$ W m^2; $c_2 = 1.44\times10^{-2}$ m K

• The temperature calculated in this way is called the brightness temperature

• This is the equivalent blackbody (ε = 1) temperature of a radiating body

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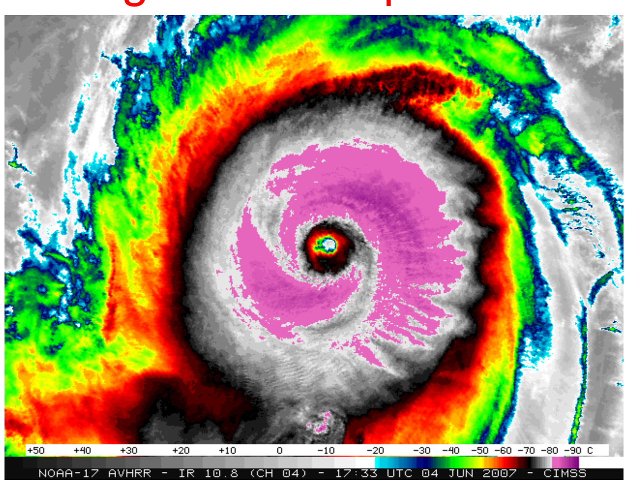
Brightness temperature



• Most land and water surfaces and dense cloud layers have emissivities of ~1 in the infrared (*note that this does not apply in the microwave region)*

• Hence the IR brightness temperature of these surfaces is close to the actual physical temperature (if viewed through a transparent atmosphere)

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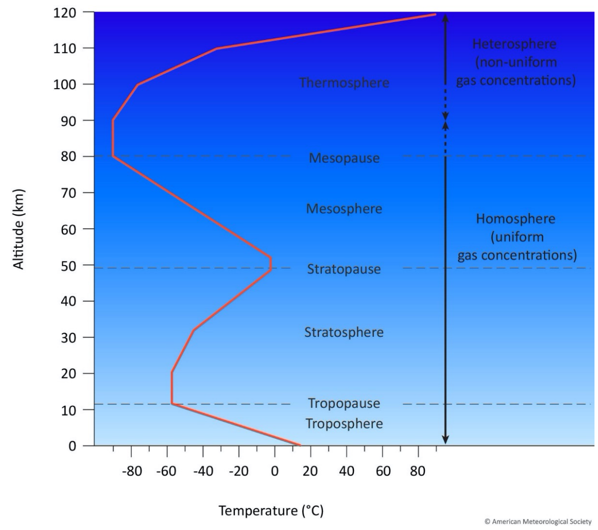
Atmospheric temperature profile



• The atmospheric temperature profile impacts the observed brightness temperature

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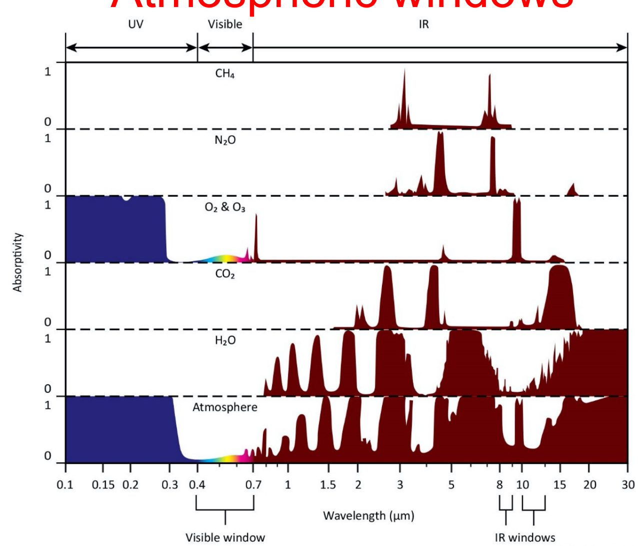
Atmospheric windows



• Variable atmospheric transmission also impacts the observed brightness temperature at a given wavelength

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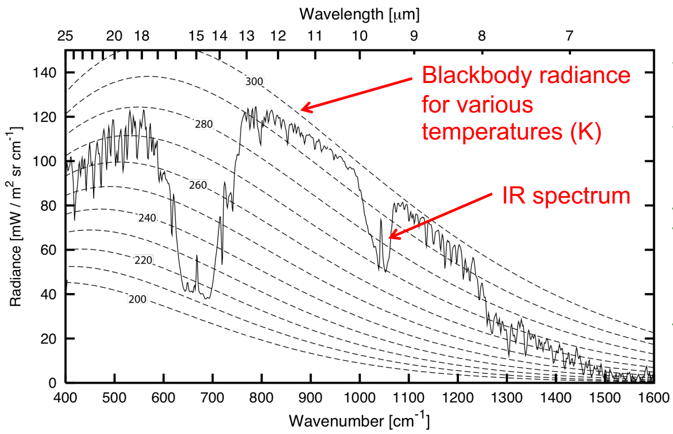
Infrared emission spectrum



What is T_B at 11 μm?

What is T_B at 15 μm?

At which of these wavelengths is the sensor primarily detecting emission from the surface?

• IR emission spectrum obtained by the Nimbus 4 satellite over the tropical Pacific Ocean on a cloudless day

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When does thermal emission matter?

• **For wavelengths shorter than a certain value** the contribution of incident and reflected solar radiation to the total radiation field far exceeds the contribution due to direct thermal emission (about 4 μm).

• **For slightly longer wavelengths**, one might need to consider both solar radiation and emission from the Earth-atmosphere system itself.

• **For even longer wavelengths**, one may often (but not always) ignore the solar component relative to the thermally emitted component

• Depends on whether irradiance (flux) or radiance is being measured

• If the latter, the viewing direction and reflectance of the surface and atmosphere are important

• 4 μm is a reasonable threshold for separating solar from thermal sources

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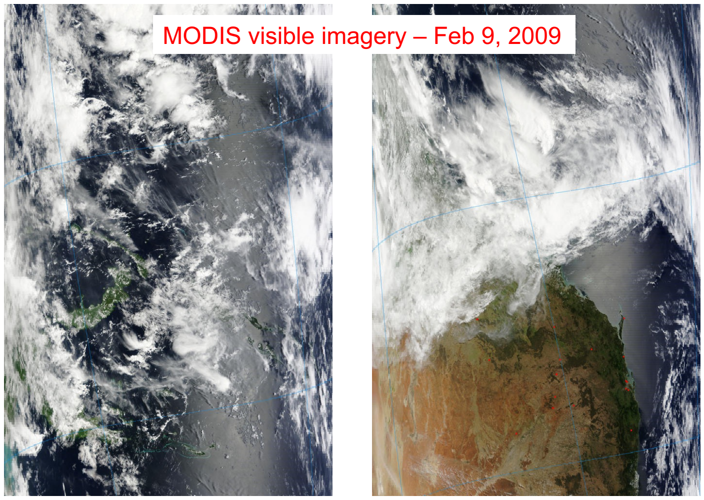
Exception: sun glint



MODIS visible imagery – Feb 9, 2009

- Observed in direction of reflection of the sun from smooth surfaces
- MODIS images: https://worldview.earthdata.nasa.gov/

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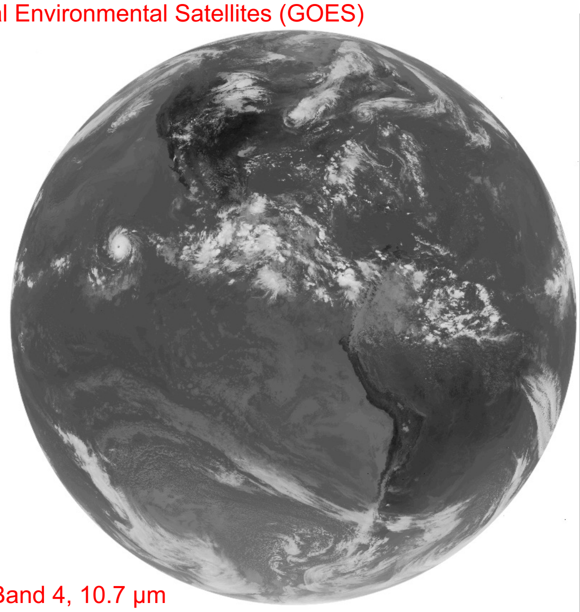
Applications: IR imaging from space

Geostationary Operational Environmental Satellites (GOES)

$$T_B = B_\lambda^{-1}\left(\varepsilon B_\lambda(T)\right)$$

- $\varepsilon \approx 1$ for most land and water surfaces, most clouds (except thin cirrus), and snow, in the thermal IR
- Hence observed brightness temperatures approximate the true physical temperature
- Important atmospheric window in the thermal IR around 11 µm
- Can estimate cloud-top altitude



Band 4, 10.7 µm

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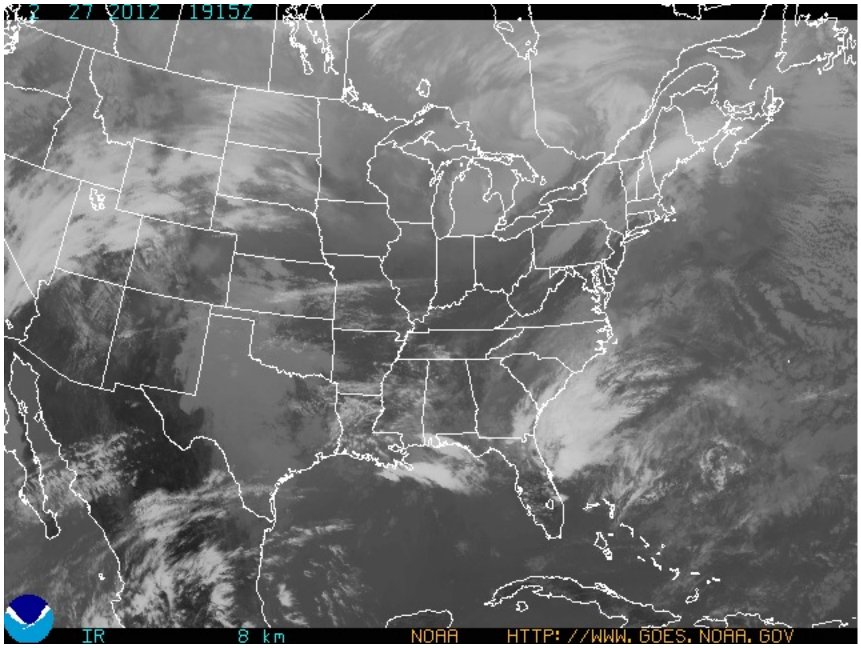

GOES Band 4 (10.7 µm) 27 Feb 2012 1915Z

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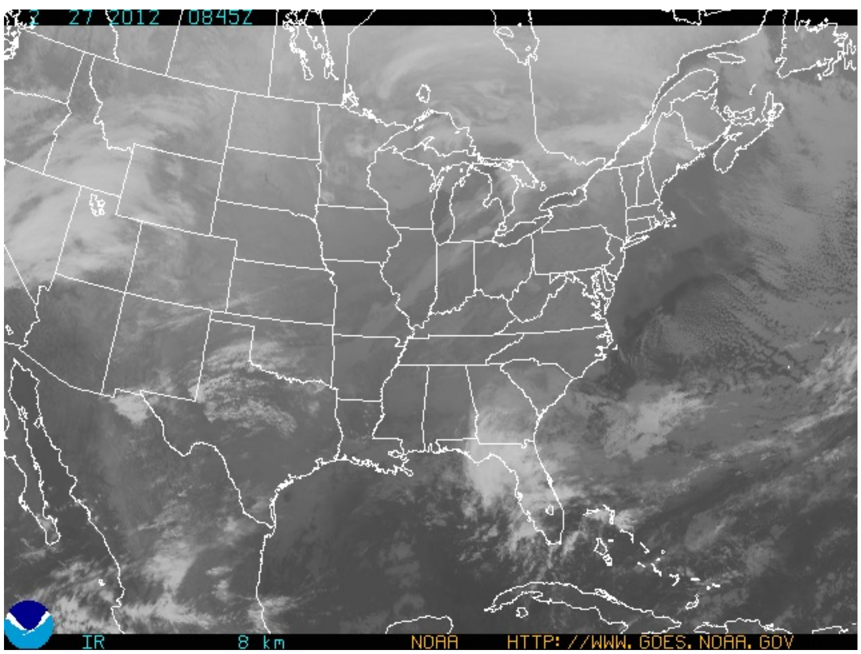

GOES Band 4 (10.7 µm) 27 Feb 2012 0845Z

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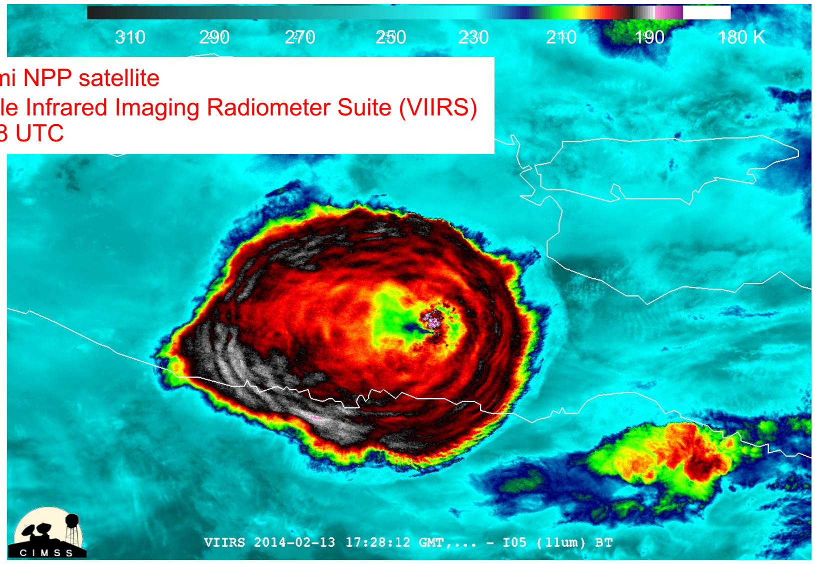
Applications: IR imaging from space



Suomi NPP satellite
Visible Infrared Imaging Radiometer Suite (VIIRS)
17:28 UTC

- Eruption of Kelut volcano, Java on Feb 13-14, 2014 at ~16:15 UTC
- Why is the top of the cloud 'warmer'?

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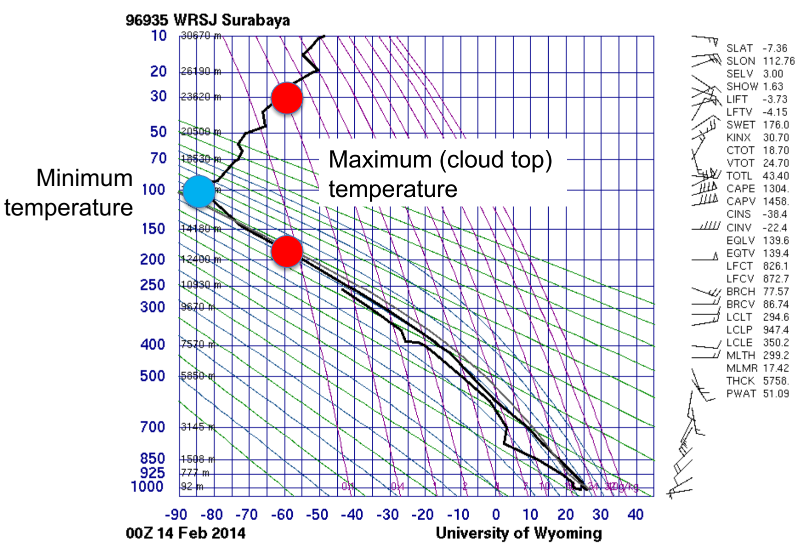
Applications: IR imaging from space



- Actual atmospheric sounding from Java on Feb 14, 2014 at 00:00 UTC
- Global soundings available here: http://weather.uwyo.edu/upperair/sounding.html

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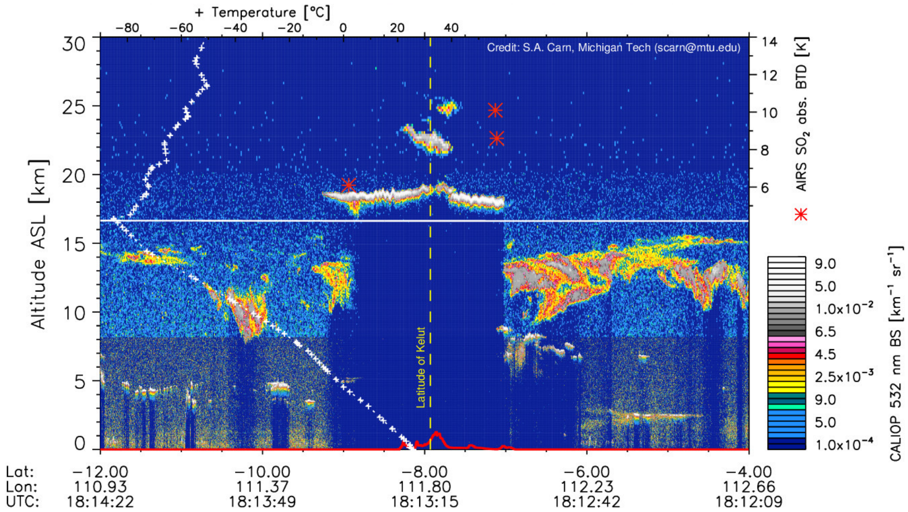
Confirmation by CALIPSO LiDAR



- Eruption of Kelut volcano, Java on Feb 13-14, 2014
- The top of the eruption cloud 'overshot' into the warmer lower stratosphere

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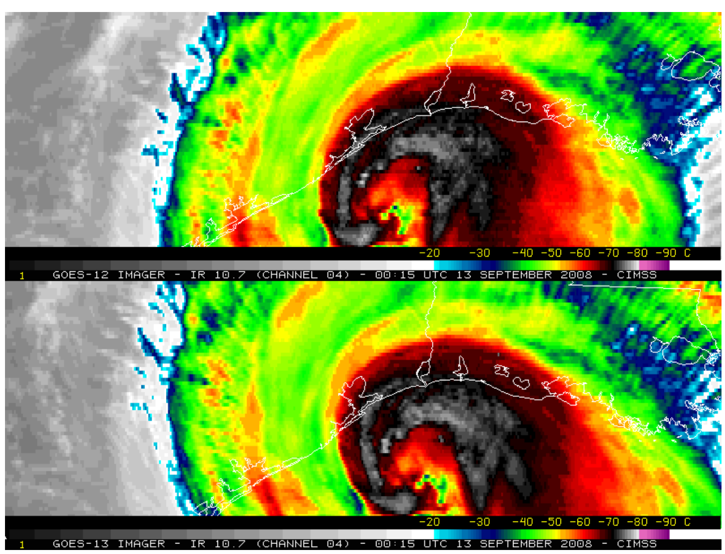
Applications: IR imaging from space

GOES
Hurricane Ike
Sept 2008



What is one advantage of using IR imagery over Visible imagery?

- Coldest scenes in IR imagery usually associated with high cirrus clouds and deep thunderstorms (e.g., in hurricanes) – used for operational tracking of severe weather

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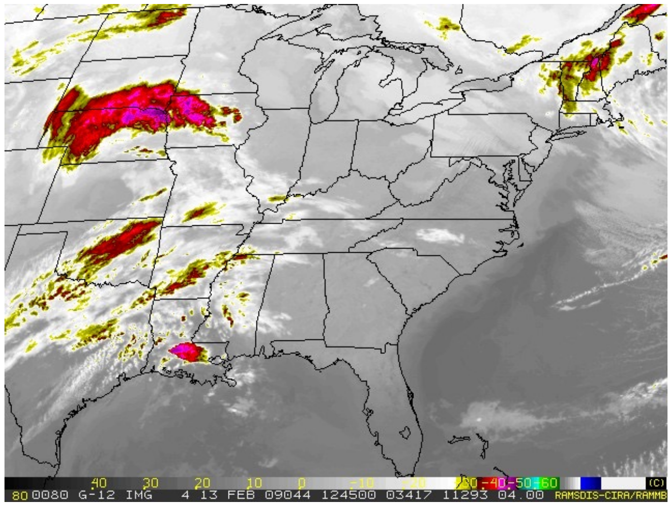
Applications: IR imaging from space



- GOES imagery: http://rammb.cira.colostate.edu/ramsdis/online/
- Comparison of visible and IR image in early morning over eastern US

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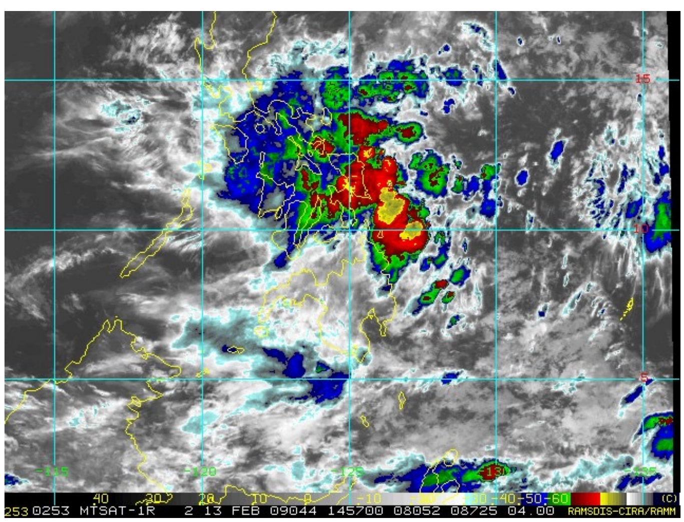
Applications: IR imaging from space



- MTSAT – Philippines and northern Indonesia
- Comparison of visible and IR image of tropical clouds

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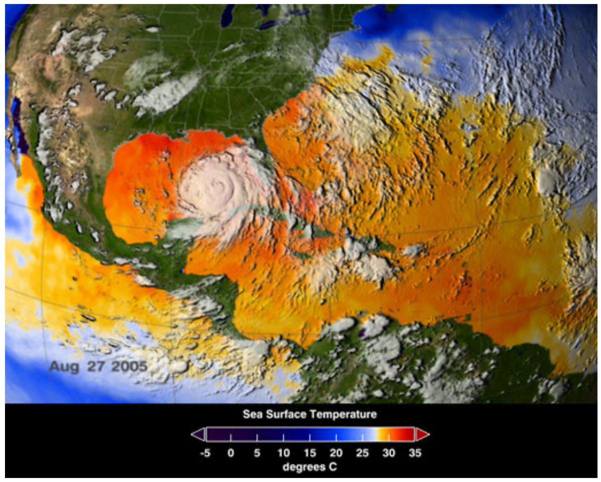
Applications: IR imaging from space



- IR imagery also the primary basis for mapping of sea surface temperatures (SSTs)
- Important for climate and weather forecasting (Gulf Stream, hurricane development, El Niño) – in-situ measurements are generally sparse

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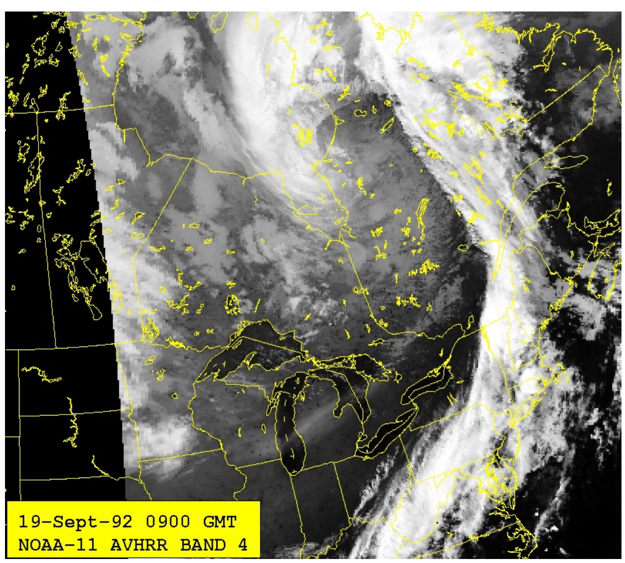
Applications: Volcanic ash detection

Mt. Spurr (Alaska) eruption – Sept 1992



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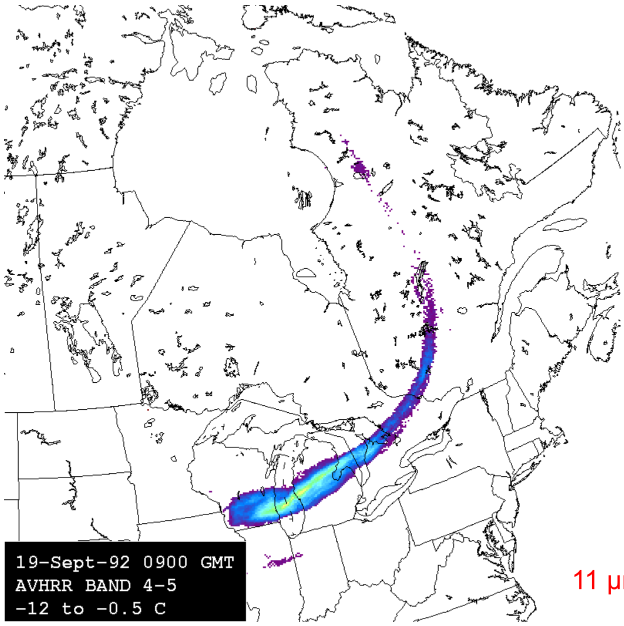
Mt. Spurr (Alaska) eruption – Sept 1992



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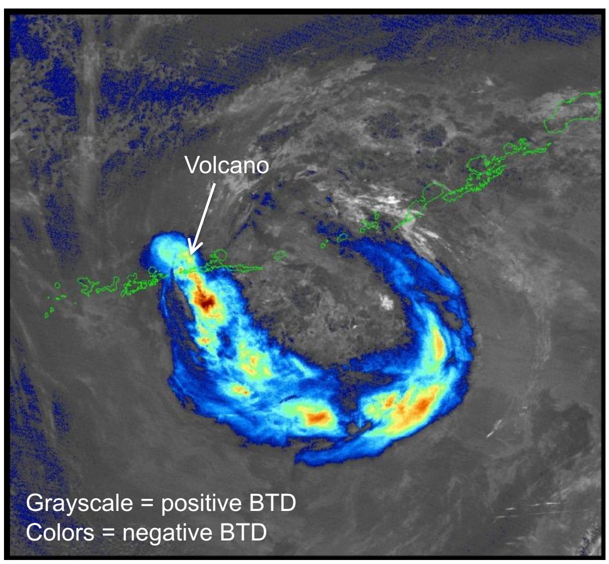
IR 'split-window' technique

AVHRR
Kasatochi volcano eruption

11-12 µm Brightness Temperature Difference (BTD)

August 2008



Volcanic ash absorbs more IR radiation at 11 µm than at 12 µm

Hence **T_B (11 µm) < T_B (12 µm)**, so the 11-12 µm BTD is negative where ash is present

- The IR split-window technique (or BTD) is also used to identify other features, such as airborne dust (e.g., sandstorms)

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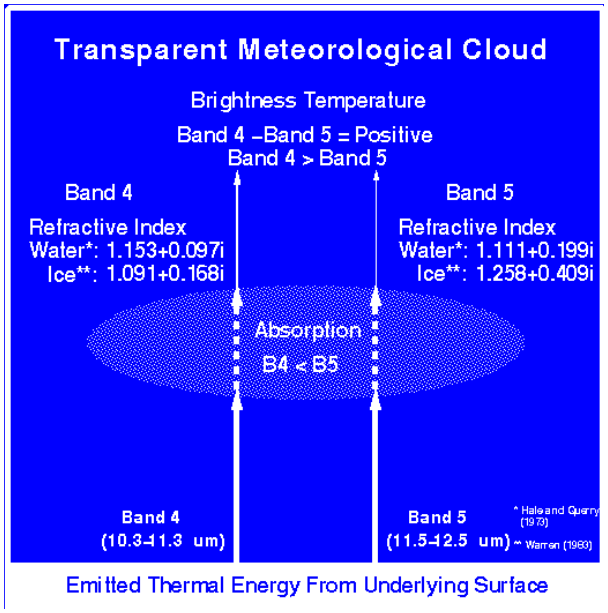
IR 'split-window' technique



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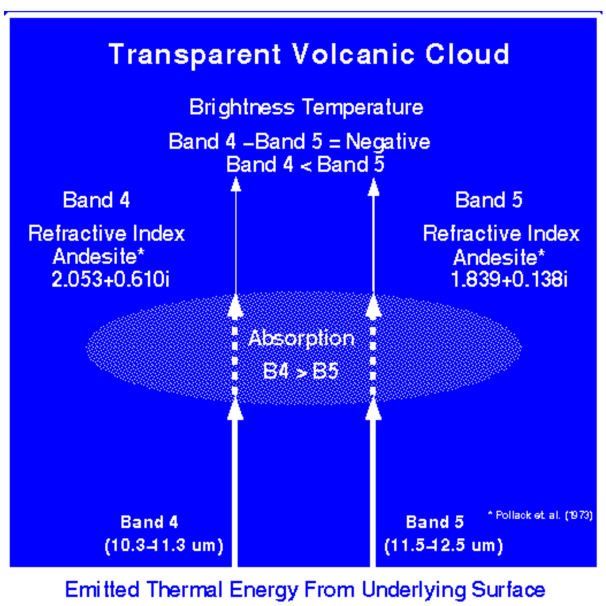
IR 'split-window' technique



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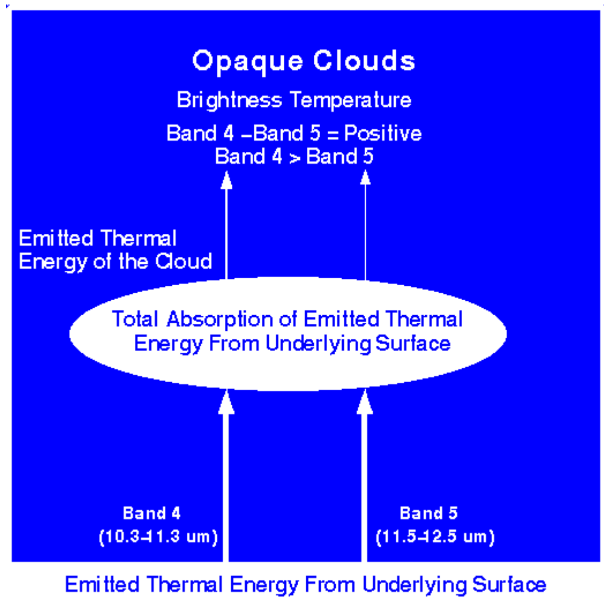
IR 'split-window' technique



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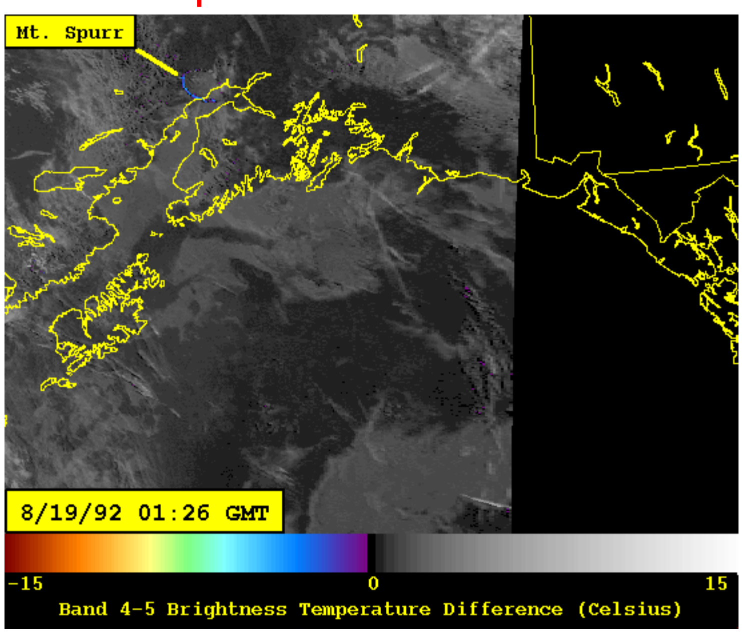
Mt. Spurr volcanic ash cloud



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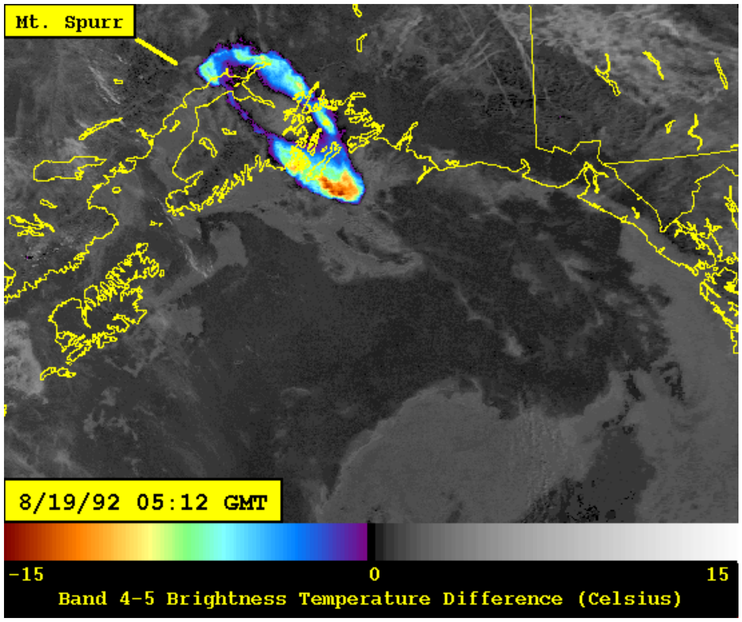
Mt. Spurr volcanic ash cloud



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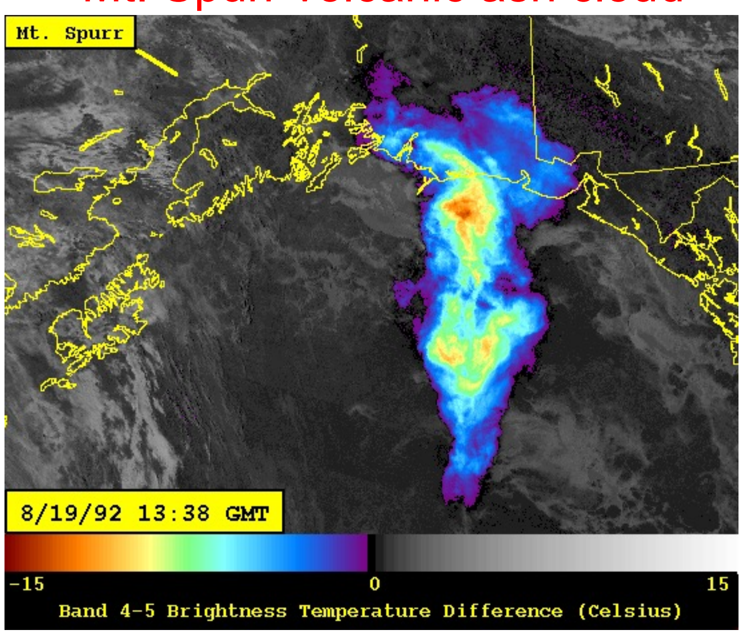
Mt. Spurr volcanic ash cloud



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Thermally complex surfaces

Lava lake at Nyiragongo volcano (DR Congo)

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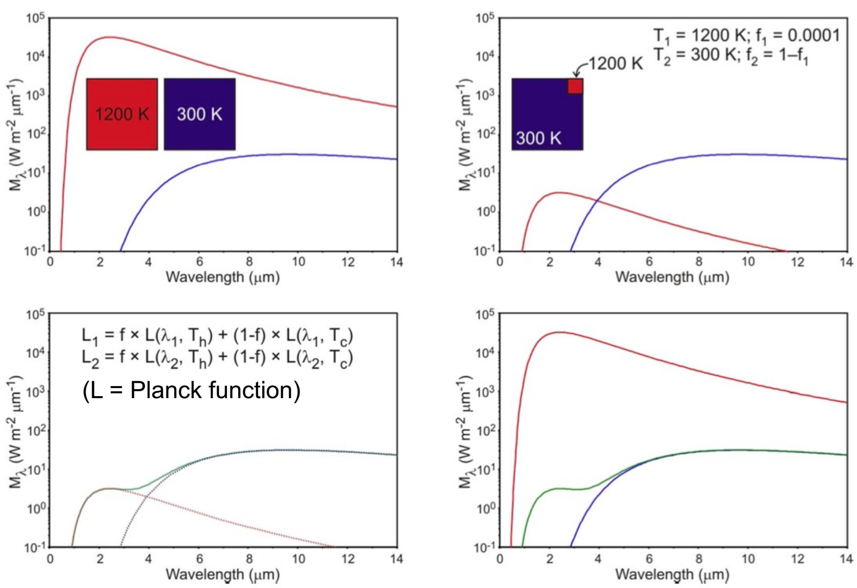
Thermal mixture modeling



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Applications: IR Thermography

• Handheld thermal imaging cameras are now widely available (e.g., https://www.flir.com/)

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Applications: IR Thermography

• Forward-Looking Infrared (FLIR) imaging (~3-5 μm, 8-13 μm)

• Many applications: volcanoes, medical, surveillance, night vision, pipeline leak detection, heat loss, sport….

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Applications: IR Thermography

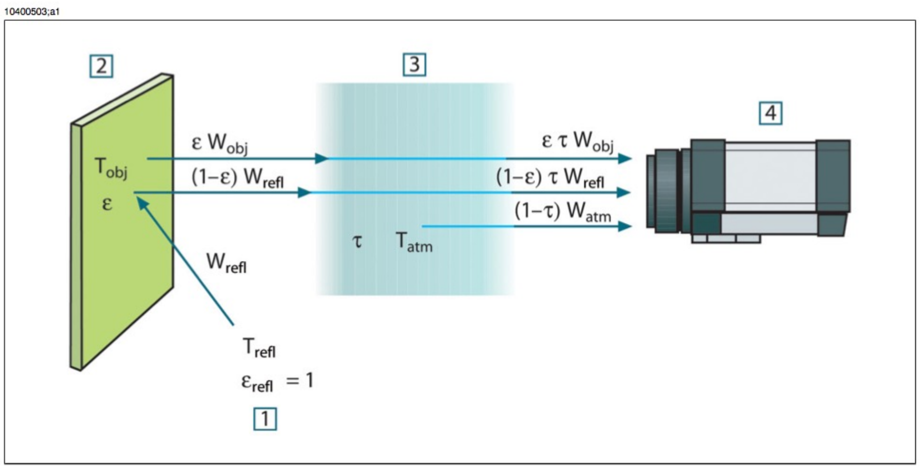

Figure 16.1 A schematic representation of the general thermographic measurement situation. **1:** Surroundings; **2:** Object; **3:** Atmosphere; **4:** Camera

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Applications: IR Thermography

$$U_{obj} = \frac{1}{\varepsilon\tau}U_{tot} - \frac{1-\varepsilon}{\varepsilon}U_{refl} - \frac{1-\tau}{\varepsilon\tau}U_{atm}$$

This is the general measurement formula used in all the FLIR Systems thermographic equipment. The voltages of the formula are:

Figure 16.2 Voltages

U_{obj}	Calculated camera output voltage for a blackbody of temperature T_{obj} i.e. a voltage that can be directly converted into true requested object temperature.
U_{tot}	Measured camera output voltage for the actual case.
U_{refl}	Theoretical camera output voltage for a blackbody of temperature T_{refl} according to the calibration.
U_{atm}	Theoretical camera output voltage for a blackbody of temperature T_{atm} according to the calibration.

The operator has to supply a number of parameter values for the calculation:

- the object emittance ε,
- the relative humidity,
- T_{atm}
- object distance (D_{obj})
- the (effective) temperature of the object surroundings, or the reflected ambient temperature T_{refl}, and
- the temperature of the atmosphere T_{atm}

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Applications: IR Thermography

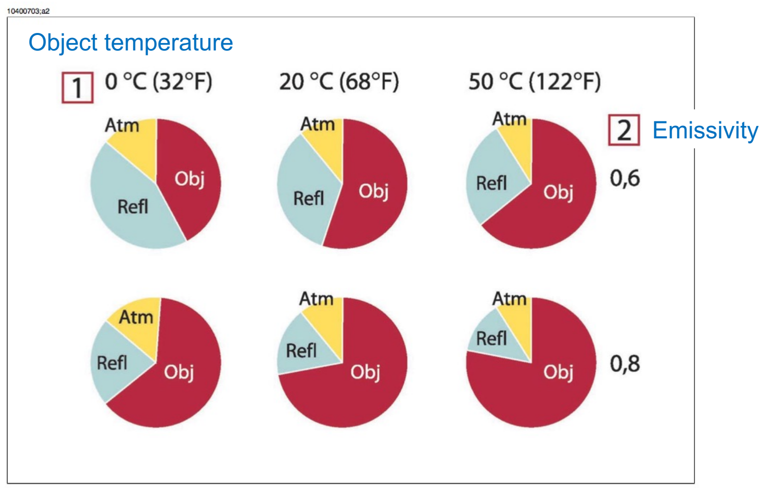

Figure 16.4 Relative magnitudes of radiation sources under varying measurement conditions (LW camera). **1:** Object temperature; **2:** Emittance; **Obj:** Object radiation; **Refl:** Reflected radiation; **Atm:** atmosphere radiation. Fixed parameters: $\tau = 0.88$; $T_{refl} = 20°C\ (+68°F)$; $T_{atm} = 20°C\ (+68°F)$.

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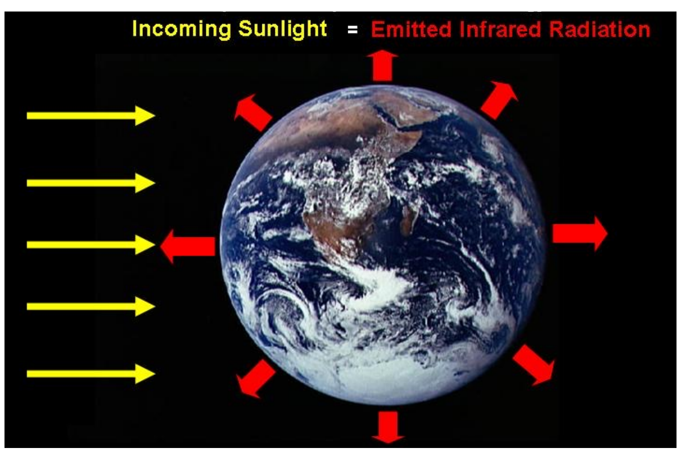
Applications: radiation balance



- Condition for radiative equilibrium: incoming shortwave radiation (from Sun) equals outgoing longwave (thermal) radiation (from Earth)
- In radiative equilibrium, average Earth temperature remains constant

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Applications: radiation balance

- Assume a single shortwave absorptivity, $\mathbf{a_{sw}}$
- Equal to (1-$\mathbf{A_{sw}}$), where $\mathbf{A_{sw}}$ is the shortwave albedo of a surface
- Assume a single longwave absorptivity, $\mathbf{a_{lw}}$
- From Kirchhoff's Law, longwave emissivity $\boldsymbol{\varepsilon} = \mathbf{a_{lw}}$
- Using the Stefan-Boltzmann Law, longwave radiation fluxes may then be computed as $\boldsymbol{\varepsilon\sigma T^4}$
- Futhermore, we can assume that $\boldsymbol{\varepsilon \approx 1}$ in the longwave band
- Shortwave emission (at Earth temperatures) is omitted altogether
- Condition for radiative equilibrium is that all fluxes balance
- If longwave (LW) emission exceeds shortwave (SW) absorption, cooling occurs and vice versa

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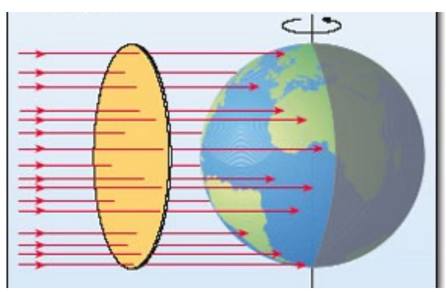
Applications: radiation balance



- Incoming SW solar radiation incident on Earth disk with area $\boldsymbol{\pi r^2}$
- Solar flux (ϕ_{SW}) given by $\mathbf{S_0(1-A_{sw})\pi r^2}$
- Where S_0 = solar constant (1370 W m^{-2}) and A_{sw} = shortwave albedo (~0.3)
- Outgoing LW emission (ϕ_{LW}) given by the product of the Stefan-Boltzmann law and the surface area:

$$\Phi_{LW} = 4\pi r^2 \sigma T_E^4$$

- T_E = equilibrium temperature (i.e. T for radiative equilibrium)

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Applications: radiation balance

• For the condition of radiative equilibrium we have:

$$\Phi_{SW} = \Phi_{LW} \Rightarrow S_0(1 - A_p)\pi r^2 = 4\pi r^2 \sigma T_E^4$$
$$\Rightarrow S_0(1 - A_p) = 4\sigma T_E^4$$
$$\Rightarrow T_E = \sqrt[4]{\frac{S_0(1 - A_p)}{4\sigma}}$$

• A_p = average planetary albedo

• Substituting the appropriate values for S_0, σ and A_p yields T_E = 255 K

• What is the actual observed global average temperature of the Earth?

• Similar calculations can be done for any planetary body

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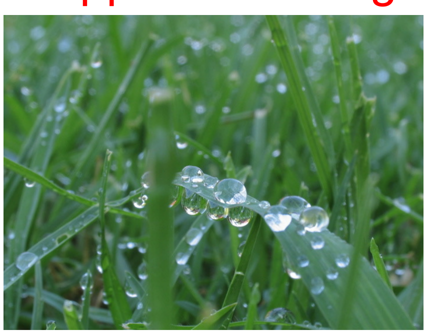
Applications: nighttime radiative cooling



• Dew or frost more likely to occur on clear nights than cloudy nights

• Requires surface to cool below the dewpoint or frostpoint temperature of the air, by radiative cooling

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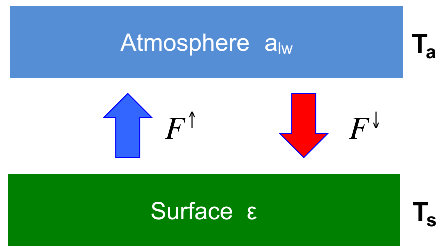
Applications: nighttime radiative cooling

Schematic of radiative coupling between the surface and a thin isothermal atmosphere



$$F^{\downarrow} = a_{lw}\sigma T_a^4$$

$$F^{\uparrow} = \varepsilon\sigma T_s^4 = \sigma T_s^4 \qquad \text{(assume } \varepsilon = 1\text{)}$$

- The cooling rate of the ground is proportional to the net flux:

$$F^{net} = F^{\uparrow} - F^{\downarrow} = \sigma T_s^4 - a_{lw}\sigma T_a^4 = \sigma(T_s^4 - a_{lw}T_a^4)$$

- For a cloud-free atmosphere, a_{lw} ranges from 0.7 (wintertime arctic) to 0.95 (tropics) – primarily controlled by water vapor
- T_a = 235 K (arctic winter) – 290 K (tropical)

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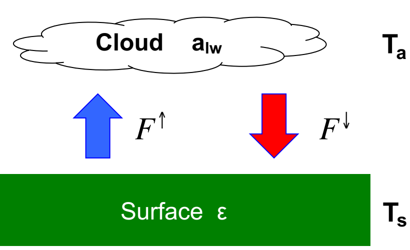
Applications: nighttime radiative cooling

- Midlatitude winter scenario: a_{lw} = 0.8, T_a = 260 K, T_s = 275 K
- Yields F^{net} = 117 W m^{-2} (positive = upwards) – hence ground surface temperature falls rapidly



- Now add low-level opaque cloud: a_{lw} = 1, T_a = 270 K, T_s = 275 K
- Yields F^{net} = 22 W m^{-2} (positive = upwards) – a much lower cooling rate than under clear sky
- Addition of greenhouse gases to the atmosphere increases a_{lw}

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