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Volcanic time-trend analysis

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Abstract

The use of nonhomogeneous Poisson processes to model volcanic events is increasingly popular, as they allow the inclusion of waxing or waning time-trends and better estimates of future activity. This model also enables the following hypotheses to be tested using simple computational algorithms: (1) Does the historical record of a single volcano show a significantly increasing (or decreasing) time-trend? (2) Is there a significant time-trend difference between two sets of volcanic recurrence interval data? (3) Does a group of volcanoes (≥ 3) show the same time-trend? The historical data of three volcanoes in New Zealand are analyzed to test each of the above hypotheses. A methodology for group model selection based on the hypothesis testing is presented.

Keywords: group model selection; homogeneous Poisson process; hypothesis testing; power-law process; volcanic hazard/risk

1. Introduction

Volcanoes and their behavior cover an enormous spectrum: from inconspicuous fissures to majestic peaks and from mild steaming to terrifying eruptive paroxysms. To understand volcanism—an essential step towards either combating its dangers or utilizing its resources—we must gauge its full breadth and attempt to wrestle its elements into some kind of framework (Simkin and Siebert, 1994). This paper is one of many efforts toward that end.

The subject of volcanic hazards has received increased attention in the past decade. In particular, the use of the nonhomogeneous Poisson process has recently gained popularity in volcanic data analysis as a simple and versatile tool to assess the waxing or waning time-trends of a volcano and to assess its volcanic hazards (see, e.g., Ho, 1990, 1995). In an earlier work (Ho, 1991), I argue that volcanoes can generally be related to a nonhomogenous Poisson process, and I propose a new method for time-trend analysis and demonstrate its usefulness with data from volcanoes Aso, Etna, Kilauea, St. Helens and Yake-Dake. The method is designed to model a single volcano (or a volcanic system treated as one point process). The main purpose of this article is to propose statistical tests for quantitative comparison of time-trends of several volcanic processes.

The article is organized into five sections: (1) notation and review of a nonhomogeneous Poisson process to model the time-trend of a single volcano; (2) an F-test for testing the similarity in trend of two volcanoes; (3) computation algorithms for comparison of more than two volcanoes; (4) the train of analyses and the numerical computations that are involved in the proposed methods using an empirical example; and (5) summary and discussion.

2. Trend analysis for one volcano

A homogeneous Poisson process assumes a constant recurrence rate, λ, for volcanic events. If the

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volcanism is waning or developing, the model should be generalized to allow λ to be, respectively, a decreasing or increasing function of t. If one replaces the constant λ with a function of t, denoted by $\lambda(t)$, then another type of Poisson process can be derived, known as a nonhomogeneous Poisson process. A nonhomogeneous Poisson process has a mean value function denoted by $\mu(t|\Theta)$, where Θ is a vector of parameters. The nondecreasing function $\mu(t|\Theta)$ represents the expected number of events in $[0,t]$. Once the functional form of $\mu(t|\Theta)$ is specified, the nonhomogeneous Poisson process is fully characterized. An alternate characterization of the nonhomogeneous Poisson process is through its intensity function $\lambda(t|\Theta)$, where:

$$\lambda(t|\Theta) = \frac{d}{dt}\mu(t|\Theta)$$

In volcanism, the intensity function $\lambda(t|\Theta)$, is the instantaneous rate of change of the expected number of eruptions with respect to time and it is called the instantaneous recurrence rate of the volcanic process. If $\lambda(t|\Theta)$ is integrated over an interval, then one obtains the expected number of eruptions in the interval. In my previous work (Ho, 1991), I let $\Theta = (\theta,\beta)$ and write:

$$\mu(t|\Theta) = (t/\theta)^{\beta}$$

so that:

$$\lambda(t|\theta,\beta) = (\beta/\theta)(t/\theta)^{\beta-1}$$

This form, termed the *power law*, has found applications in reliability analysis due to its flexibility (in the sense that the intensity function can be constant, decreasing, or increasing). A noteworthy feature of my approach is that by replacing the expected number of events in a homogeneous Poisson process, λt, with $\mu(t) = (t/\theta)^{\beta}$, I let the volcanic data determine the time-trend for themselves: increasing ($\beta >$ 1), decreasing ($\beta < 1$), or random ($\beta = 1$ which assumes a no-memory property). To model the volcanic time-trend using a power-law process, let t be predetermined and suppose $n > 1$ eruptions are observed during $[0,t]$ at time $0 < t_1 \le t_2 \le \dots \le t_n \le t$. Some useful theoretical results to be used later are summarized as follows:

(1) Let $S = \sum_{i=1}^{n} \ln(t/t_i)$, then the maximum likelihood estimators of β and θ are given (Crow, 1974) by:

$$\hat{\beta} = n/S$$

$$\hat{\theta} = t/n^{1/\hat{\beta}}$$

(2) Under the null hypothesis $H_0{:}\beta = 1$, $2S \sim \chi^2$ $(2n)$. Therefore, a size α test of $H_0{:}\beta = 1$ against $H_A{:}\beta \neq 1$ is to reject H_0 if $2S \le \chi^2_{\alpha/2}$ $(2n)$ or $2S \ge \chi^2_{1-\alpha/2}$ $(2n)$, where $\chi^2_{\alpha/2}$ $(2n)$ is the $100\alpha/2$ percentile of a chi-square distribution with $2n$ degrees of freedom.

(3) If a power-law process is assumed during the observation time period $[0,t]$, the intensity (instantaneous recurrence rate) is $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ at time t. In the application of the power-law process to volcanic eruptive forecasting, the estimate of $\lambda(t)$ is of considerable practical interest because $\lambda(t)$ represents the instantaneous eruptive status of the volcanism at the end of the observation time t. Crow (1982) derives the maximum likelihood estimator for $\lambda(t)$ as:

$$\hat{\lambda}(t) = \left(\hat{\beta}/\hat{\theta}\right)\left(t/\hat{\theta}\right)^{\hat{\beta}-1} = n\hat{\beta}/t$$

Clearly, the power-law process generalizes the homogeneous Poisson process, because when $\beta = 1$ the power-law process reduces to a homogeneous Poisson process. The chi-square test defined in (2) provides a quantitative method to objectively evaluate whether the time-trend of the volcanic activities during the observation period (a) remains approximately Poissonian, or (b) shows a significantly increasing (or decreasing) time-trend. I note that in a simulation study, Bain et al. (1985) conclude that the chi-square test which is derived as an optimal test for the power-law process also is rather powerful as a test of trend for general nonhomogeneous Poisson processes. In other words, the test is "robust" against other model assumptions. This is the rationale of choosing a power-law process to model volcanic eruptions.

For relatively short time intervals, or where the data do not indicate any trend, point processes other than Poisson may also prove useful for modeling inter-event times. Bebbington and Lai (1996a) recently proposed a Weibull renewal process as a model for volcanoes with no apparent overall time-trend. A renewal process is a sequence of random variables $\{Y_1, Y_2, \ldots\}$ of the form $Y_n = X_1 + \ldots + X_n$, where $\{X_1, X_2, \ldots\}$ are independent and identically distributed with distribution $F(X)$. Thus a renewal process model for a volcanic system conjectures that the volcano returns to the same starting conditions after each eruption. This assumption is reasonable when no time-trend is evident. This form of behavior would be analogous to sequentially using light bulbs with the same lifetime distribution (e.g., an exponential or a Weibull distribution) in one socket with instantaneous replacement at failure.

3. F-test for testing similarity of two volcanoes

If data are obtained from a single volcano and inferences are made only for that volcano, then a power-law process with fixed values of the parameters is an appropriate model. However, there are many situations in which more than one volcano is involved in a simple exploratory analysis. For example, Klein (1982) compares repose times for differences between large and small, summit and flank, and Kilauea and Mauna Loa eruptions.

Engineers are able to compare several repairable systems based on statistical methods. Volcanic eruptions are individually unique, but volcanism as a whole is a nonunique process in which repeated combinations of rate balances give rise to categorically similar patterns worldwide. Given sufficiently redundant information, pattern recognition and comparisons with the observed patterns become automatic. Later, I demonstrate a generalized method of quantitative description and comparisons of the volcanic processes.

Suppose now that independent volcanic repose time series of sizes n_1 and n_2 are observed, and two power-law processes with shape parameters β_1 and β_2 are assumed, respectively, for each process. Let S_1 and S_2 be the corresponding statistics as described in (1), then the overall time-trends of these two volcanic processes can be quantitatively compared using the following test.

(4) Let $F = n_2 S_1 / n_1 S_2$, then under the null hypothesis $H_0: \beta_1 = \beta_2$, $F \sim F(2n_1, 2n_2)$. And, a size α test of $H_0: \beta_1 = \beta_2$ against $H_A: \beta_1 \neq \beta_2$ is to reject H_0 if $F \leq F_{\alpha/2}(2n_1, 2n_2)$ or $F \geq F_{1-\alpha/2}(2n_1, 2n_2)$, where $F_{\alpha/2}(2n_1, 2n_2)$ is the $100\alpha/2$ percentile of an F-distribution with $2n_1$ and $2n_2$ degrees of freedom. Dot plots showing the visible time-trends of the volcanoes in the empirical studies section will demonstrate the usefulness of the F-test.

4. Test statistic for more than two volcanoes

Suppose k (>2) volcanoes are observed for a fixed length of time, t, and volcano i has n_i eruptions at successive time $0 < t_{i1} \leq t_{i2} \leq \ldots \leq t_{in_i} \leq t$. Again, some useful theoretical results to be used later are summarized as follows:

(5) Let $S_i = \sum_{j=1}^{n_i} \ln(t/t_{ij})$, then the maximum likelihood estimator of β_i derived by Engelhardt and Bain (1987) is:

$$\hat{\beta}_i = n_i / S_i$$

which has the same form as the maximum likelihood estimator of the parameter β, as described in (1), for a single power-law process model. Also, the useful relationship to the chi-square distribution for $\hat{\beta}_i$, in the power-law process case carries over to the case with more than one power-law process. Namely, the chi-square test described in (2) is also applicable for testing $H_0: \beta_i = 1$ against $H_A: \beta_i \neq 1$ for any $i = 1, 2, \ldots, k$.

(6) A test of equality of shape parameters, $H_0: \beta_1 = \beta_2 = \ldots = \beta_k$ rejects this hypothesis at the approximate level α if:

$$M \geq c\chi^2_{1-\alpha}(k-1)$$

where:

$$c = 1 + \frac{1}{3(k-1)}\left(\sum_{i=1}^{k} \frac{1}{2n_i}\right) - \frac{1}{2N}$$

$$M = 2N\left[\ln\left(\sum_{i=1}^{k} n_i/\hat{\beta}_i\right) - \ln N\right] + 2\sum_{i=1}^{k} n_i \ln(\hat{\beta}_i)$$

and: $N = \sum_{i=1}^{k} n_i$, the total number of eruptions for all k volcanoes.

Interested readers are referred to the article of Engelhardt and Bain (1987) for theoretical development and further references. In the next section, I apply these computation algorithms to volcanic data to produce informative time-trend analyses.

5. Empirical example

5.1. Data

Simkin et al. (1981) constructed a chronology of known volcanic events over the past 8000 yr. The record has been updated through December 31, 1993 (Simkin and Siebert, 1994). The eruption records (adopted from Volcanoes of the World, 2nd edition, Simkin and Siebert, 1994) of the following three volcanoes in New Zealand are studied for time-trend analyses: White Island, Tongariro and Ruapehu. Please note that Tongariro is treated as one volcano, and not as a volcanic center consisting of several volcanoes, the principal one being Mt. Ngauruhoe. This follows the convention of Simkin and Siebert (1994) who group all Tongariro events under one listing. Since Mt. Ngauruhoe accounts for nearly all observed events in Tongariro, the results should not be changed much by considering only activity at Mt. Ngauruhoe. Please note that Bebbington and Lai (1996b) also used the occurrence data of Mt. Ruapehu and Mt. Ngauruhoe to test their Weibull renewal model, a suggested alternative stochastic process.

The record of volcanic activity analyzed in this article has the form of a point process (i.e., a record of the month and year during which each eruption occurred). Several simplifying assumptions must be made in treating eruptions as a point process in time: (1) Although the onset date of an eruption is generally well defined by the time when lava first breaks the surface or when ash is first ejected, the duration is harder to determine because of such problems as slowly cooling flows or lava lakes and the gradual decline of explosive activity. I therefore adopt the definition for repose time proposed by Klein (1982) and ignore eruption duration. Instead, the onset date is considered physically meaningful, and repose times are measured from one onset date to the next. Thus, my definition of "repose time" differs from the classic one (a noneruptive period). This simplifying procedure seems justified by two reasons, that most eruption durations are much shorter than the typical repose intervals (Klein, 1982), and because the historical data often lack any reliable account of actual eruption duration. Each data set of a power-law process consists of the cumulative length of time (measured in months) over which the eruptions occur. (2) On several occasions, the months during which eruptions occurred are uncertain and were therefore assigned somewhat arbitrarily. (3) The first recorded eruption of White Island volcano was on the first day of December 1826. Therefore, this date becomes my choice of the starting point for the observation period for all three volcanoes. The last day of year 1993 is the end of the observation period, which is the same as that of the listed volcanoes in Simkin and Siebert (1994).

5.2. Time-trend analyses

To be consistent with the mathematical notations presented in (1) through (6), I label volcanoes White Island, Tongariro and Ruapehu as volcano no. 1, 2 and 3, respectively, for the following analyses and discussions.

(1) During the observation period, December 1, 1826 to December 31, 1993, the data for the number of recurrence intervals are $n_1 = 32$, $n_2 = 70$ and $n_3 = 50$. The estimated shape parameters for the time-trend are $\hat{\beta}_1 = 1.913$, $\hat{\beta}_2 = 1.305$ and $\hat{\beta}_3 = 3.516$ (see Table 1). The result implies that all three volcanoes show an increasing trend (i.e., $\beta > 1$) during the observation period. The two-sided p-values summarized in Table 1 indicate that Tongariro volcano provides only moderate evidence against H_0 ($\beta_2 = 1$) with p-value $= 0.037$, while the other two volcanoes show strong evidence against H_0. Dot

Table 1
Summary statistics for trend analysis

	Volcano		
	White Island	Tongariro	Ruapehu
n_i	32	70	50
S_i	16.725	53.647	14.219
$\hat{\beta}_i$	1.913	1.305	3.516
$\hat{\lambda}_i$ (no. of eruptions/month)	0.031	0.046	0.088
Chi-square test statistic (for $H_0: \beta_i = 1$)	33.450	107.293	28.438
p-value (two-tailed)	0.001	0.037	≈ 0

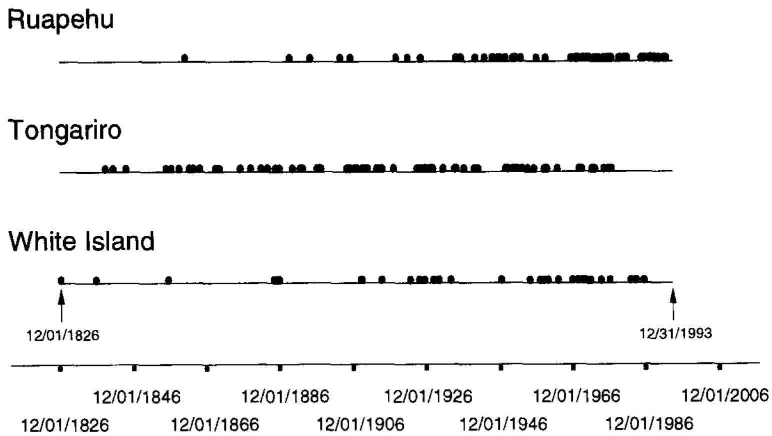

Fig. 1. Dot diagrams of recurrence intervals (in months) of volcanoes Ruapehu, Tongariro and White Island in their original chronological orders observed from December 1, 1826 to December 31, 1993.

diagrams presented in Fig. 1 reconfirm the quantitative results. (Note that it is common practice to describe the location of the observed result in the null distribution of a test statistic by giving the tail area or tail probability beyond the observed value. This probability is called the p-value. The smaller it is, the farther the observed value is from the expected value when H_0 is true, and the harder it is to accept the discrepancy as sampling variability. Thus, a very small p-value is evidence against the null hypothesis. This is purely a subjective matter. Many statisticians take the following interpretations as benchmarks: $p < 0.01$—strong evidence against H_0; $0.01 < p < 0.05$—moderative evidence against H_0; $p > 0.1$—little or no evidence against H_0. There is a strong tradition that arbitrarily selects the values 0.01 and 0.05 as critical levels for p-values, using this language: when $p < 0.05$, the result is called *statistically significant*; and when $p < 0.01$, the result is called *highly statistically significant*. There seems to be a common perception that the 0.01 and 0.05 critical levels have a theoretical basis, but they are in fact arbitrary.)

(2) The instantaneous recurrence rate estimated on December 31, 1993 for Mount Ruapehu ($\hat{\lambda}_3 = 0.088$/month) is higher than that of Tongariro ($\hat{\lambda}_2 = 0.046$/month), although Tongariro volcano produced twenty more eruptions than Mount Ruepehu during the same observation period. Because the power-law process incorporates the time-trend, evidence of additional events will not necessarily increase the instantaneous recurrence rate as it would for the homogeneous Poisson process recurrence rate. This noteworthy feature of the power-law process model is of considerable practical interest in volcanic risk/hazard assessment studies (e.g., see Ho, 1995).

(3) For the pairwise comparisons, let's consider volcanoes White Island and Tongariro: $n_1 = 32$, $n_2 = 70$, $S_1 = 16.725$, $S_2 = 53.647$ (see Table 1) and the degrees of freedom for the F-distribution are $2n_1 = 64$ and $2n_2 = 140$. The test does not reject H_0: $\beta_1 = \beta_2$ because $F = 0.682$ and the two-tailed p-value is 0.086 (see Table 2). Thus, we consider that the shape parameter, β, is statistically the same for volcanic activities of White Island and Tongariro volcanoes during the observation period. However,

Table 2
Results of F tests for pairwise comparisons

	White Island vs. Tongariro	White Island vs. Ruapehu	Tongariro vs. Ruapehu
F-statistic	0.682	1.838	2.695
p-value (two-tailed)	0.086	0.006	≈ 0

comparisons (see Table 2 and Fig. 1) between Tongariro versus Ruapehu, and White Island versus Ruepehu show that the differences are significant with p-values 0.006 and ≈ 0, respectively. Note that these still stand up nicely to a Bonferroni corrected alpha of $0.05/3 = 0.01\overline{6}$, if one chooses to do the adjustment of alpha for multiple tests. (A procedure that provides a family level of significance, α, is often highly desirable since it permits the analyst to weave the separate results together into an integrated set of conclusions, with an assurance that the entire set of hypotheses is correct. The Bonferroni method of developing multiple tests with a specified family level of significance is a very simple one: the individual level of significance is adjusted to be lower than α so that the family level of significance is at most α. The method is a general one that can be applied in many cases.)

(4) Now, recall from (6) of the previous section that a test of equality of shape parameters, $H_0: \beta_1 = \beta_2 = \beta_3$ rejects this hypothesis at the approximate level 0.05 if:

$$M \geq c\chi^2_{0.95}(2)$$

For this study, I get $c = 1.002$, $\chi^2_{.95}(2) = 5.99$, and the critical value $c\chi^2_{0.95}(2)$ is approximately 6.002. Because the test statistic M is 26.359, H_0 is rejected at $\alpha = 0.05$ as I have expected from the previous results of pairwise comparisons. Actually, the test is significant at any level since the p-value is ≈ 0. Therefore, I conclude that these volcanoes do not share a common shape parameter, β, which serves as an indicator for the time-trend of the volcanic activities. It has been suggested that differences in the shape parameter may be related to broad geological distinctions between volcanoes. Although an interesting conjecture, there is currently insufficient evidence to comment on this.

(5) Finally, what are the merits of performing the above tests? I shall discuss this issue based on the following scenarios that one might conclude from the trend analyses.

Case 1: $\beta_1 = \beta_2 = \beta_3 = 1$

Case 2: $\beta_1 = \beta_2 = \beta_3 = \beta \neq 1$

Case 3: $B_i \neq \beta_j$ for some i, j, where $1 \leq i < j \leq 3$

For Case 1, a compound homogeneous Poisson process can be used to model the aggregate behavior of these Poissonian volcanoes ($\beta = 1$). In a compound homogeneous Poisson process model, the recurrence rate for a given volcano or group of volcanoes is described by a gamma distribution (prior) rather than treated as a constant value as in the assumptions of a homogeneous Poisson process. I performed Bayesian analysis (Ho, 1990) to link these two distributions together to give the aggregate behavior of the volcanic activity. When the homogeneous Poisson process is expanded to accommodate a gamma mixing distribution on λ, a consequence of this mixed (or compound) Poisson model is that the frequency distribution of eruptions in any given time period of equal length follows the negative binomial distribution. Applications of the model and comparisons between this generalized model and a homogeneous Poisson process were discussed based on the historical eruptive count data of volcanoes Mauna Loa and Etna (Ho, 1990). Where several relevant facts led to the conclusion that the generalized model is preferable for practical use both in space and time. A similar situation can occur with a group of non-Poissonian volcanoes ($\beta \neq 1$). If one replaces the underlying distribution in a compound homogeneous Poisson process with a nonhomogeneous Poisson process distributed according to a power-law process and also let the intensity parameter vary according to a gamma distribution as described in the model of Ho (1990), then a new model called compound power-law process provides a better fit than a compound homogeneous Poisson process. Statistical analysis of a compound power-law process for repairable systems has been presented in an article by Engelhardt and Bain (1987). This model requires several additional assumptions, discussion of which is beyond the scope of this paper. In future studies, I expect to develop the volcanological aspect of a common power-law process and further refine the potential usefulness of this model in volcanology. For Case 3, to my best knowledge, a single model such as a compound homogeneous Poisson process for Case 1 and a compound power-law process for Case 2 is not available.

6. Conclusions

Volcanic activity is governed by the complex interaction of several geological, geophysical and geochemical factors. Because of this complexity, even with the present knowledge, eruptions cannot theoretically be predicted. Therefore, the evaluation of eruptive probabilities for a given volcano or a volcanic center remains an open problem in the definition of volcanic risk. There are many unknown areas with respect to geologic understanding of volcanic activity, despite the fact that there are well recognized means of gathering data (field mapping, determination of the eruptive histories of volcanic centers, petrology, geochemistry, geochronology and geophysical studies) that are well advanced. Also, developments in short term volcanic prediction, such as the Materials Failure Forecast Method (Cornelius and Voight, 1995), present understanding of eruptive mechanisms is not yet advanced enough to allow deterministic predictions of long term future activity. Analysis of volcanic hazard/risk over greater periods of time (anywhere from years to millenia) remains outside the scope of current deterministic models. The only attempts at long-term forecasting have been made on statistical grounds, using historical records to examine eruption frequencies, types, patterns, risks and probabilities. Furthermore, as noted by Simkin and Siebert (1994), the detection and determination of volcanic events is subject to both psychological influences and technical developments. Both the selection of historical data and the interpretation of the statistical analysis should be done with these facts in mind. Nevertheless, statistical time-trend analysis retains its usefulness as a quantitative analytical tool which can be applied to any volcano with sufficiently complete data. Should sufficient information exist about a volcano, perhaps the data can be partitioned and analyzed separately to account for the better monitoring of recent activity. This paper extends my previous work (Ho, 1991) on testing the significance of increasing or decreasing time-trends of volcanoes. I now add two new test statistics to the geological literature which represents a good cross-application of statistics to geosciences. In summary, the significance of this work is: quantitative comparisons between (or among) volcanoes become possible and a clear-cut guideline for volcanic model selection process evolves.

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