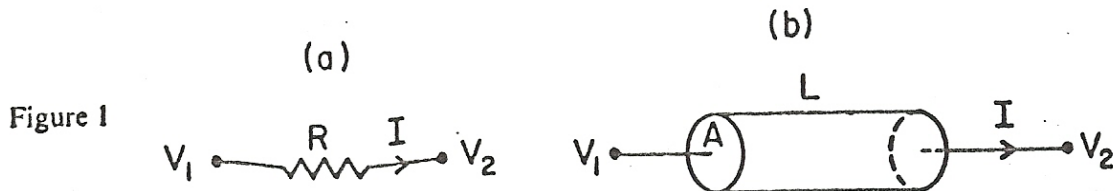


ELECTRICAL RESISTIVITY VALUES FOR EARTH MATERIALS

1. Resistivity versus resistance

Most people are familiar with electrical resistance through Ohm's Law, $V = R I$. As shown in Figure 1 (a), a DC (direct current) voltage difference $V = V_1 - V_2$ across the ends of resistance R will cause a current I to flow.



Resistivity is related to resistance but is not identical to it. To understand the difference, consider a cylinder as shown in Figure 1(b). If a DC voltage difference V causes a current I to flow, then the resistance of the cylinder can be measured experimentally as

$$R = V/I$$

Further experiments can be used to demonstrate that R depends upon the length of the cylinder, L , the cross sectional area, A , and a characteristic property of the cylinder material, ρ , as

$$R = \rho \frac{L}{A}$$

The quantity ρ is called electrical resistivity. We note that it is a bulk property of material, analogous to density, whereas resistance depends in addition upon the geometry of the sample. Since the units of resistance are ohms, we conclude that the units of resistivity must be ohms times length, for example ohm-meters.

2. Factors which control the resistivity of earth materials

For DC current flow such as used in resistivity exploration, the electrical current is carried through the earth material by either (a) motion of free electrons or ions in the solid, or (b) motion of ions in the connate water. Item (a) may be occasionally important when dealing with certain kinds of minerals such as graphite, magnetite, or pyrite. For most applications in engineering and hydrogeology, however, item (b) is overwhelmingly dominant.

The ions in the connate water come from the dissociation of salts such as sodium chloride, magnesium chloride, etc. We find that, for water-bearing rocks and earth materials, the resistivity decreases with increasing

- (a) fractional volume of the rock occupied by water,

- (b) salinity or free-ion content of the connate water,
- (c) interconnection of the pore spaces (permeability),
- (d) temperature.

Factors (a) and (b) may be incorporated into an empirical relationship known as Archie's Law. For saturated porous sandstones and limestones, the bulk resistivity, ρ , depends upon the porosity, ϕ , and the resistivity of the connate water, ρ_w , as

$$\rho = a \rho_w \phi^{-b}$$

where a is a number of the order of 1 and b is a number of the order of 2.

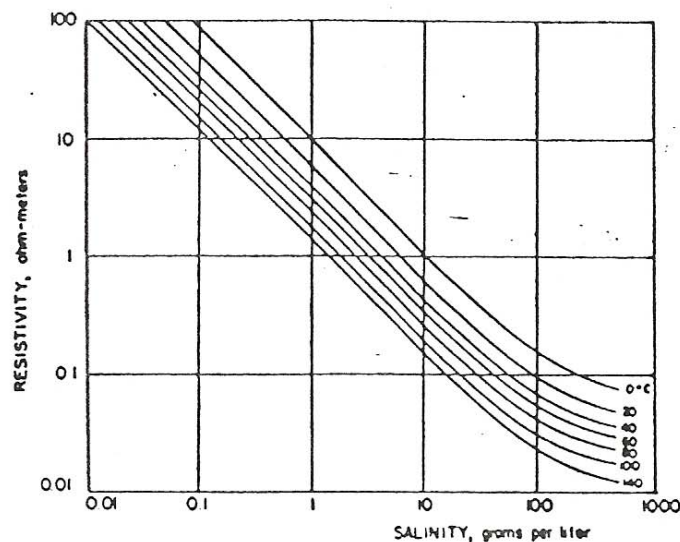


Figure 2: Dependence of electrical resistivity upon salinity and temperature (Keller and Frischknecht, 1966)

The dependence upon salinity and temperature is shown in Figure 2. We observe that a temperature increase from 0° to 100°C produces a five-fold reduction in resistivity, for any level of salinity. This has direct relevance to geothermal prospecting by resistivity methods. The total temperature effect will be even greater than this, since increasing temperature can increase the solubility factor as well as the mobility factor.

From the preceding, we may infer that

- (a) Materials which lack pore space will show high resistivity such as
 - massive limestone
 - most igneous and metamorphic rocks such as granite, basalt
- (b) Materials whose pore space lacks water will show high resistivity such as
 - dry sand or gravel
 - ice

- (c) Materials whose connate water is clean (free from salinity) will show high resistivity such as
 - clean gravel or sand, even if water saturated
- (d) Most other materials will show medium or low resistivity, especially if clay is present, such as
 - clay soil
 - weathered rock.

The dependence upon water saturation is not linear. The resistivity may increase relatively little as saturation decreases from complete to the range of 40-60% and then increase much more rapidly as saturation continues to decrease.

Frozen ground shows much higher resistivity than unfrozen ground, but the effect enters gradually rather than abruptly as the temperature falls from 0° to -10° or -15°. Figure 3 illustrates the behaviour.

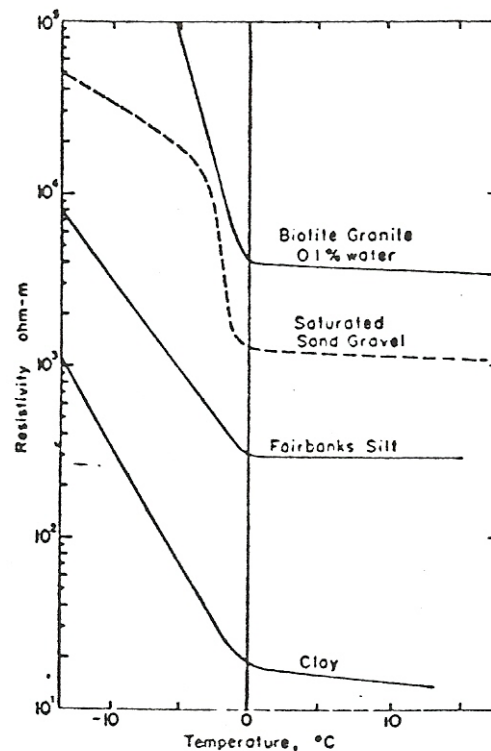


Figure 3: Dependence of electrical resistivity upon temperature in frozen soil and rock (Hockstra, Sellman, and Delaney, 1975)

The presence of clay minerals tends to decrease the resistivity because (a) the clay minerals can combine with water, (b) the clay minerals can adsorb cations in an exchangeable state on the surface, and (c) the clay minerals tend to ionize and contribute to the supply of free ions.

3. Representative resistivity values

As a rough guide, we may divide earth materials into

— low resistivity	less than 100 ohm meters
— medium resistivity	100 to 1000 ohm meters
— high resistivity	greater than 1000 ohm meters

Some representative resistivity values are:

Regional soil resistivities

— wet regions	50-200 ohm meters
— dry regions	100-500
— arid regions	200-1000 (sometimes as low as 50 if the soil is saline)

Waters

— soil water	1 to 100
— rain water	30 to 1000
— sea water	order of 0.2
— ice	10^5 to 10^8

Rock types below the water table

— igneous and metamorphic	100 to 10,000
— consolidated sediments	10 to 1000
— unconsolidated sediments	1 to 100

Ores

— massive sulfides	10^{-4} to 1
— non-metallics (gypsum, quartz, dry rock salt)	order of 10^{10}

Effect of water salinity

.005 g/liter	1050
.10	110
.5	12

REFERENCES

1. Hoekstra, P., P. Sellman, and A. Delaney, 1975, Ground and airborne resistivity surveys of permafrost near Fairbanks: Geophysics, Volume 40, p. 641.
2. Keller, G. V., and F. C. Frischknecht, 1966, Electrical Methods in Geophysical Prospecting Pergamon Press, New York, 517 pp.
3. Orellana, E., 1972, Prospeccion Geoelectrica en Corriente Continua: Paraninfo, Madrid 523 pp.

Electrode Configurations

Conventions:

A = current electrode (+)

B = " " (-)

M = potential electrode (1)

N = " " (2)

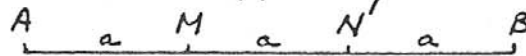
a = distance between successive electrodes

\overline{AB} = distance between current electrodes

\overline{MN} = distance between potential electrodes

O = center point of array

I Wenner Array



$$V_{P_1} = \frac{\rho I}{2\pi} \left(\frac{1}{a} - \frac{1}{2a} \right)$$

$$V_{P_2} = \frac{\rho I}{2\pi} \left(\frac{1}{2a} - \frac{1}{a} \right)$$

$$\Delta V = V_{P_1} - V_{P_2} = \frac{\rho I}{2\pi} \left(\frac{1}{a} - \frac{1}{2a} - \frac{1}{2a} + \frac{1}{a} \right)$$

$$\Delta V = \frac{\rho I}{2\pi a}$$

$$\rho = 2\pi a \Delta V / I$$

II Lee-Partitioning Array

A $\underbrace{\quad a \quad M \quad \alpha_1 \quad 0 \quad \alpha_2 \quad N \quad a \quad}_{\quad} \quad B$

$$V_{P_1} = \frac{\rho I}{2\pi} \left(\frac{1}{a} - \frac{1}{2a} \right)$$

$$V_{P_0} = \frac{\rho I}{2\pi} \left(\frac{1}{1.5a} - \frac{1}{1.5a} \right) = 0$$

$$\Delta V = V_{P_1} = \frac{\rho I}{4\pi a}$$

$$\rho = 4\pi a \Delta V / I$$

III Schlumberger Array

A $\underbrace{\quad M \quad N \quad}_{\quad} \quad B$

$$\overline{AB} \gg 5MN$$

$$V_{P_1} = \frac{\rho I}{2\pi} \left(\frac{1}{\overline{AB}/2 - \overline{MN}/2} - \frac{1}{\overline{AB}/2 + \overline{MN}/2} \right)$$

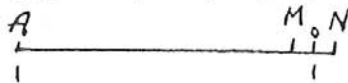
$$V_{P_2} = \frac{\rho I}{2\pi} \left(\frac{1}{\overline{AB}/2 + \overline{MN}/2} - \frac{1}{\overline{AB}/2 - \overline{MN}/2} \right)$$

$$\Delta V = V_{P_1} - V_{P_2} = \frac{\rho I}{2\pi} \left(\frac{(\overline{AB}/2 + \overline{MN}/2)(\overline{AB}/2 - \overline{MN}/2) - (\overline{AB}/2 - \overline{MN}/2)(\overline{AB}/2 + \overline{MN}/2)}{(\overline{AB}/2)^2 - (\overline{MN}/2)^2} \right)$$

$$\Delta V = \frac{\rho I}{\pi} \left(\frac{\overline{MN}}{(\overline{AB}/2)^2 - (\overline{MN}/2)^2} \right)$$

$$\rho = \pi \left[\frac{(\overline{AB}/2)^2 - (\overline{MN}/2)^2}{\overline{MN}} \right] \Delta V / I$$

III Half Schlumberger Array



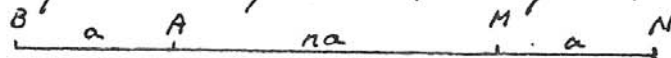
$$V_{P_1} = \frac{\rho I}{2\pi} \left(\frac{1}{AO/2 - MN/2} \right); V_{P_2} = \left(\frac{1}{AO/2 + MN/2} \right) \frac{\rho I}{2\pi}$$

$$\Delta V = \frac{\rho I}{2\pi} \left(\frac{AO/2 + \frac{MN}{2} - AO/2 + \frac{MN}{2}}{(AO/2)^2 - (MN/2)^2} \right)$$

$$\Delta V = \frac{\rho I}{2\pi} \left(\frac{MN}{(AO/2)^2 - (MN/2)^2} \right)$$

$$\rho = 2\pi \left[\frac{(AO/2)^2 - (MN/2)^2}{MN} \right] \Delta V / I$$

V Dipole - Dipole Array



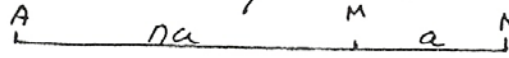
$$V_{P_1} = \frac{\rho I}{2\pi} \left(\frac{1}{na} - \frac{1}{a(n+1)} \right)$$

$$V_{P_2} = \frac{\rho I}{2\pi} \left(\frac{1}{a(n+1)} - \frac{1}{a(n+2)} \right)$$

$$\begin{aligned} \Delta V = V_{P_1} - V_{P_2} &= \frac{\rho I}{2\pi} \left(\frac{1}{na} - \frac{1}{a(n+1)} - \frac{1}{a(n+1)} + \frac{1}{a(n+2)} \right) \\ &= \frac{\rho I}{2\pi} \left(\frac{(n+1)(n+2) - n(n+2) - n(n+2) + n(n+1)}{na(n+1)(n+2)} \right) \end{aligned}$$

$$\rho = \pi a \frac{\Delta V}{I} n(n+1)(n+2)$$

VI Pole-Dipole Array



$$V_{P_1} = \frac{\rho I}{2\pi na}$$

$$V_{P_2} = \frac{\rho I}{2\pi a(n+1)}$$

$$\Delta V = \frac{\rho I}{2\pi} \left(\frac{n+1-n}{na(n+1)} \right)$$

$$= \frac{\rho I}{2\pi} \left(\frac{1}{na(n+1)} \right)$$

$$\rho = 2\pi an(n+1) \Delta V / I$$

Some Notes on Field Methods

1. Electrical Profiling

For profiling, the Wenner array is recommended either in the "broadside" or "in-line" configuration. The recommended value of "a" equals the depth of interest multiplied by a factor of approximately $1\frac{1}{2}$ -2. This value will be large enough to detect the structure of interest, yet not so large that the total volume of earth under measurement becomes excessive. A factor of 1 would be marginal. If the profile is repeated with different values of "a", a factor of $1\frac{1}{2}$ -1 can be used to emphasize shallower variations.

2. Vertical Electrical Sounding

For sounding, the Schlumberger array is recommended. Successive electrode spacings should be approximately equally spaced on a logarithmic scale. The interval between data points will depend upon the number of data points per decade. (One decade equals a factor of 10.) To achieve six data points per decade, each value for electrode spacing must equal the previous value multiplied by $10^{1/6} = 1.47$. For example, if the smallest electrode spacing ("a" for Wenner or "AB/2" for Schlumberger) equals 1 meter, then successive values would be 1.47, 2.15, 3.16, 4.64, 6.81, 10.00, 14.68,...etc. In practice, one may choose to round these off slightly to more convenient values. Three data points per decade is considered sparse and somewhat marginal.

To sound to a desired depth of investigation D, the recommended range of electrode spacings ("a" for Wenner or "AB/2" for Schlumberger) extends from a minimum of D/5 to a maximum of 4-6 times D. These figures should be regarded as minimal, suitable for qualitative or semi-quantitative interpretation. For quantitative interpretation, the data should span at least 2 decades and preferably $2\frac{1}{2}$ to 3 decades.

3. Special Requirements for Schlumberger Measurements

Schlumberger sounding must always be carried out under the constraint that MN is small compared to AB/2. A common guideline is that MN must never exceed 20% AB/2.

The field procedure consists in expanding AB/2 while holding MN fixed. This process yields a rapidly decreasing potential difference across MN, which ultimately exceeds the measuring capabilities of the instrument. At this point, a new value for MN must be established, typically 2-4 times larger than the preceding value, and the survey is continued. The last one or two AB/2 values should be duplicated with the new MN-value. The same process may need to be repeated later.

To illustrate, we suppose that the survey starts with MN = 0.3 meter. AB/2 values are taken as AB/2 = 1.00, 2.15, 3.16, 4.64, 6.81, 10.0, 14.7. At AB/2 = 14.7, we suppose that the instrument sensitivity has declined such that a larger value of MN is called for. We increase it to MN = 1.0 meter, repeat the last two AB/2 values, and continue: AB/2 = 10.0, 14.7, 21.5, 31.6, 46.4, 68.1. At AB/2 = 68.1, we suppose that we must again change MN, to 3.0 meters, say. The process continues in this way until the survey is completed.

The change in MN-values during the progress of the sounding introduces a problem for interpretation, but also an unexpected benefit. The problem arises because the apparent resistivity values turn out to differ slightly for the same $AB/2$ value when MN is changed. The results would of course be identical if the earth were truly uniform, but this will rarely be true.

The effect is shown in Figure 1. For a downgoing segment of the resistivity sounding curve, the new value for apparent resistivity will be larger than the old value. For an upgoing segment, the new value will be smaller than the old value.

For interpretation, the segmented curve shown in Figure 1 must be converted to a single smooth curve. The process is shown by the dotted lines. The smooth curve follows the right-hand portion of each segment. The justification is that the ratio, $AB/2MN$, will be largest along the right-hand portions, thus approaching more closely to the ideal of an infinite ratio. The remainder of the dotted line is drawn below the data points for a downgoing segment, and above the data points for an upgoing segment. The final dotted curve provides the field curve for interpretation.

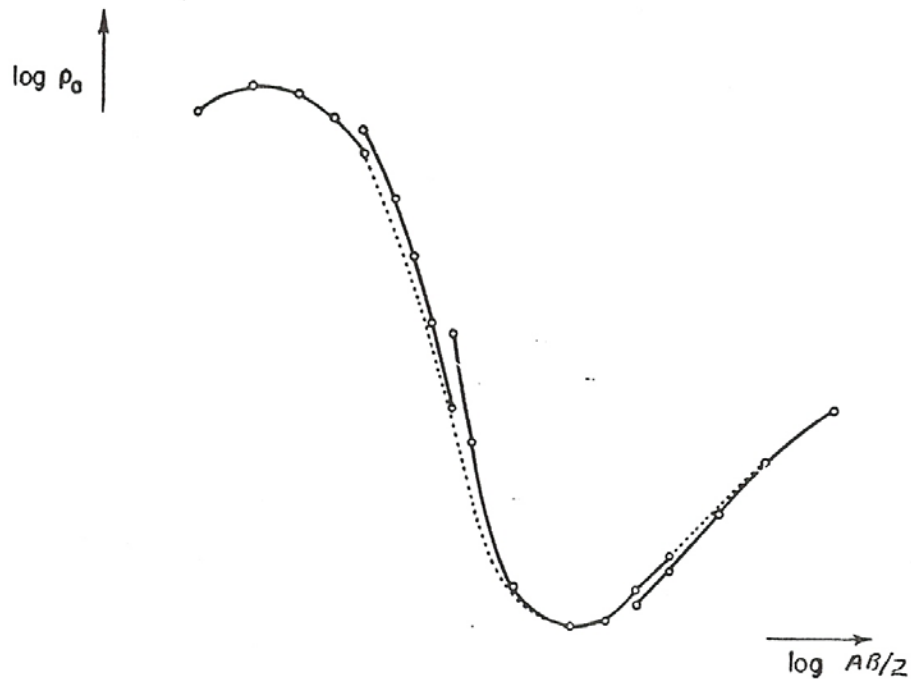


Figure 1. Effect of MN changes on the Schlumberger resistivity sounding curve.

Figure 2 shows a set of field data (Zohdy, 1974) which have been corrected to yield a single sounding curve for interpretation.

The unexpected benefit arises because the data can also reveal the presence of lateral resistivity variations. Remember the interpretation of resistivity sounding data assumes that the resistivity varies only with depth, not with horizontal position. If the subsurface does indeed contain unknown horizontal variations, these can alter the readings and produce incorrect interpretations.

The offsets in Figure 1 will be produced as shown if the resistivity varies only with depth. The presence of lateral variations will produce different onsets, possibly even reverse offsets. Thus if the field data show such anomalous offsets, the presence of lateral variations in resistivity is revealed. The amount of the anomaly may be used to estimate the importance of such lateral variations.

4. Electrode Resistances Should be Kept Low, so Far as Possible

Typically, the electrode resistance will be a few thousand ohms. Larger values will decrease the instrument sensitivity and may introduce spurious potentials. High resistance at the current electrodes will appear as low total current flow. High resistance at the potential electrodes will appear as low sensitivity and ambiguity in taking the potential reading.

The following steps may be used to reduce electrode resistance: (1) Drive the electrodes down to moist earth if possible. In some areas this may be a few centimeters and in other areas a meter or more. (2) Pour water or salt water around the electrodes. (3) Use non-polarizing electrodes. Such an electrode consists of a porcelain jar with a porous unglazed bottom. The jar is filled with a metallic electrode of the same metal. Calomel or copper/copper sulfate are commonly used.

5. Instrumentation Problems and Considerations

(1) If the potential measuring circuit draws any current, a polarization voltage may develop at the contact between the potential electrodes and the soil; this will appear as a spurious voltage in series with the true voltage; (2) if the potential electrodes are metallic, electrochemical potentials may arise due to interaction with soil fluid; (3) natural earth currents may be flowing past the electrodes, producing extraneous natural potentials which add to the desired artificial potentials. To counteract (1) the potential measuring circuit must have input impedance which is much greater than the sum of the resistances of the potential electrodes. The use of low frequency alternating current overcomes difficulties (2) and (3) as well as the use of porous pots (described above) for (2).

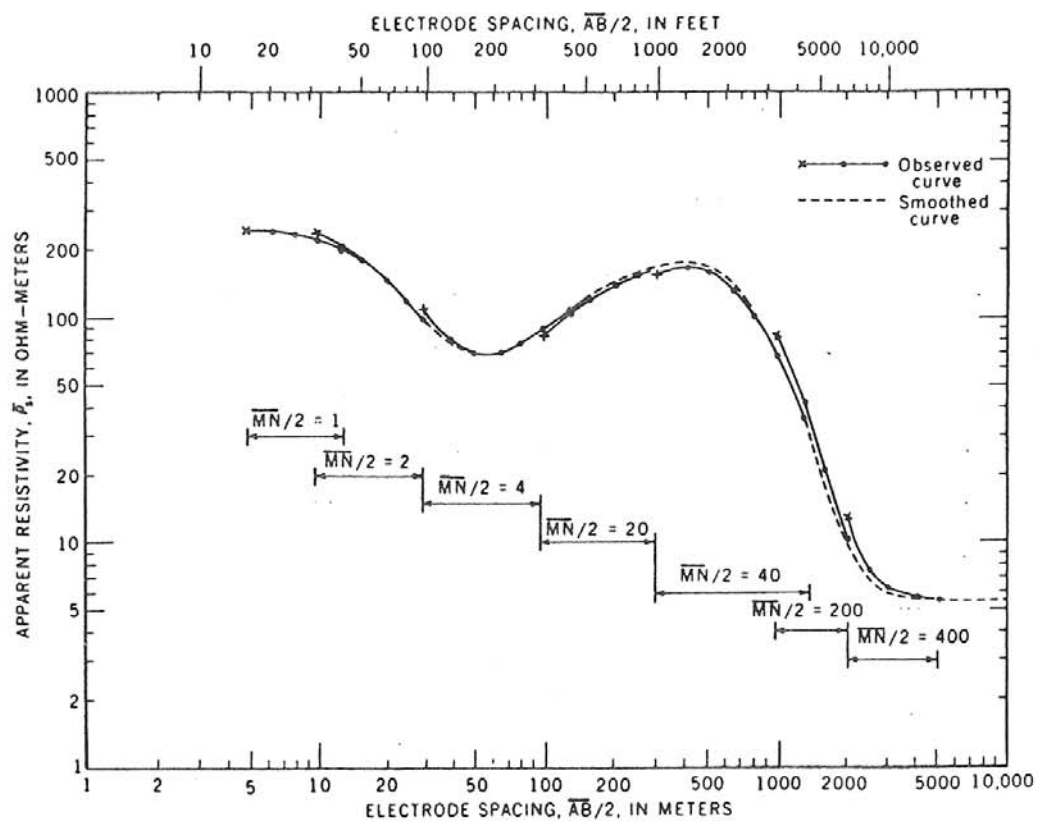


Figure 2

Advantages of Using Logarithmic Coordinates

Electrical sounding data should be plotted on logarithmic coordinates with the electrode spacing on the abscissa and the apparent resistivity on the ordinate. The advantages of plotting the sounding data on logarithmic coordinates are:

1. Field data can be compared with pre-calculated theoretical curves for given Earth models (curve-matching procedure).
2. The form of an electrical sounding curve does not depend on the resistivity and thickness of the first layer provided that the ratios $\frac{p_2}{p_1}, \frac{p_3}{p_1}, \dots, \frac{p_n}{p_1}$ and the ratios $\frac{h_2}{h_1}, \frac{h_3}{h_1}, \dots, \frac{h_n}{h_1}$ remain constant from model to model, where $p_1, p_2, p_3, \dots, p_n$ are the resistivities and $h_1, h_2, h_3, \dots, h_n$ are the thicknesses of the first, second, third, and n^{th} layers, respectively. When the absolute values of p and h change but the ratios $\frac{p_i}{p_1}$ and $\frac{h_i}{h_1}$ where $i = 2, 3, \dots, n$, remain constant, the position of the curve is merely displaced vertically for changes in p , and horizontally for changes in h . Consequently, two curves with different values of p , and h , (but with the same values of $\frac{p_2}{p_1}$ and $\frac{h_2}{h_1}$) can be superposed by translating one curve on top of the other (while the ordinate and abscissa axes remain parallel). This is the essence of the curve-matching method. Furthermore, in the computation of theoretical sounding curves the thickness and resistivity of one of the layers can be assumed equal to unity, which eliminates two parameters in the calculation of a sounding curve for a given Earth model. When sounding curves are plotted on linear coordinates, the form, as well as the position, of the curve varies as a function of p_1 and h_1 , even when the ratios $\frac{p_2}{p_1}$ and $\frac{h_2}{h_1}$ remain constant.
3. The use of logarithmic coordinates, on the one hand, suppresses the effect of variations in the thickness of layers at large depths, and it also suppresses variations of high resistivity values. On the other hand, it enhances the effect of variations in the thickness of layers at shallow depths, and it enhances the variations of low resistivity values. These properties are important because the determination of the thickness of a layer to within ± 10 meters (± 32.8 feet) when that layer is at a depth of several hundred meters is generally acceptable, whereas a precision to within one meter is desirable when the layer is at a depth of only a few tens of meters. Similarly, the determination of the resistivity of a conductive layer (less than about 20 ohm-m) to the nearest ohm-m is necessary for determining its thickness accurately, whereas for a resistive layer (more than about 200 ohm-m), the determination of its resistivity to within one ohm-m is unimportant.
4. The wide spectrum of resistivity values measured under different field conditions and the large electrode spacings necessary for exploring the ground to moderate depths make the use of logarithmic coordinates a logical choice.

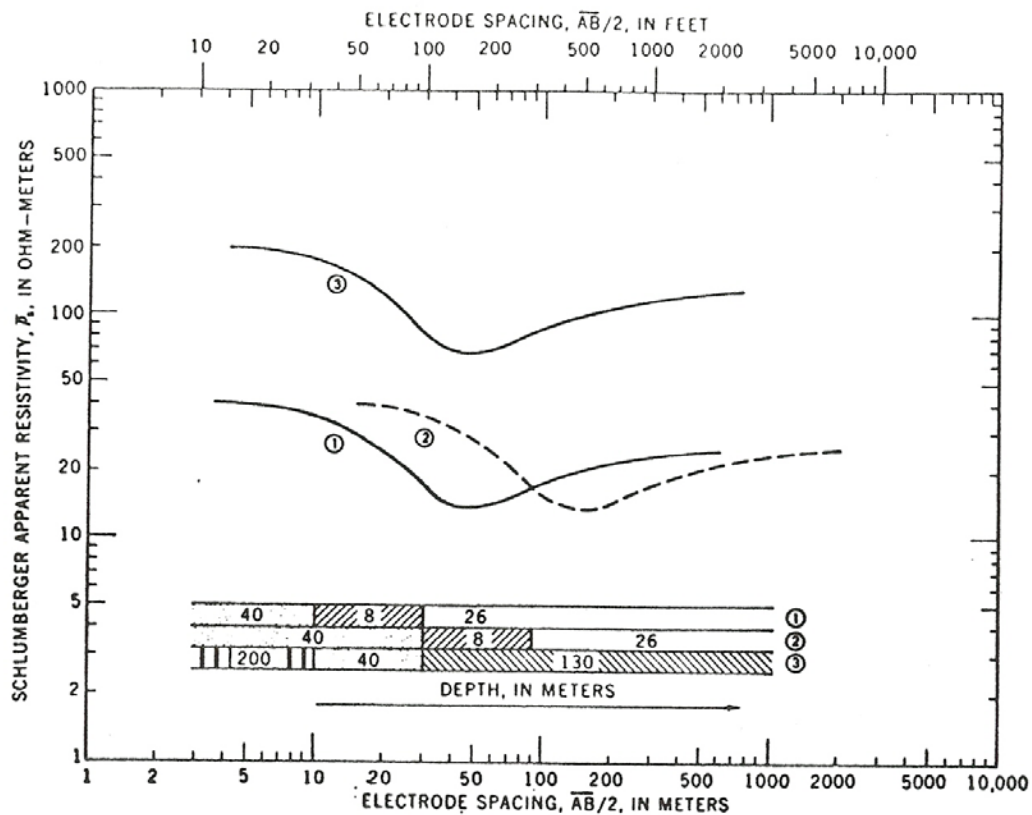


Figure 13.—Logarithmic plot of sounding curves. The layers in model 2 are three times as thick as model 1; the layer resistivities in model 3 are five times as large as model 1; however, the shapes of all three curves are identical.

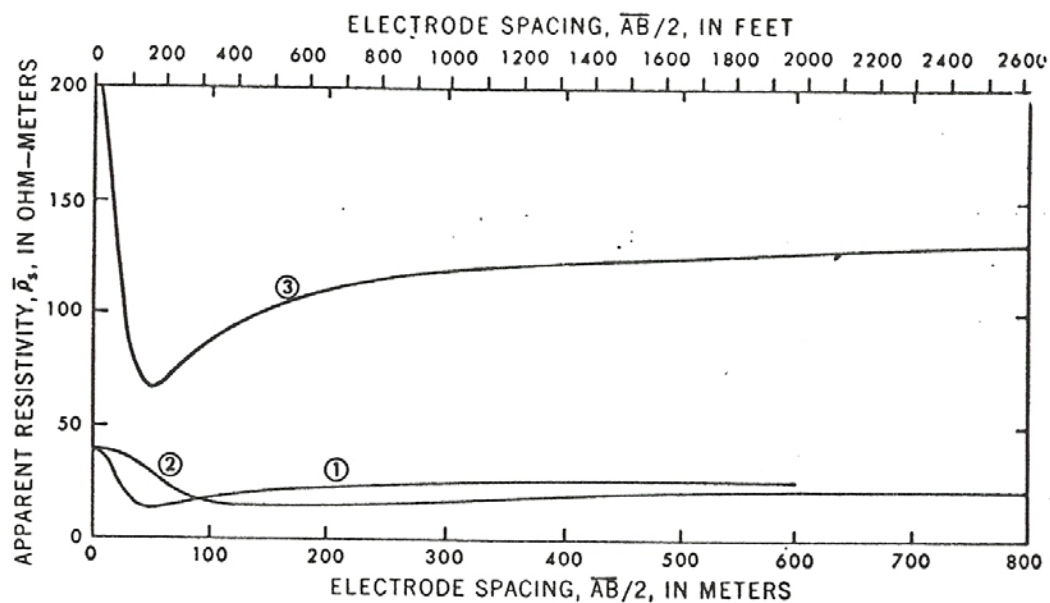


Figure 14.—Linear plot of sounding curves. Earth models are the same as in figure 13. Curve form is not preserved.

Curve Matching

The first method for quantitative interpretation is called curve matching. A graph of the field readings is matched against a theoretical graph which has been computed for particular layer resistivities. If a match can be obtained, then the subsurface structure is assumed to be identical with the theoretical structure.

The process can be best carried out using double-logarithmic paper. The reason is that the shape and size of the theoretical curves becomes independent of the units of measurements for this type of paper so that any one theoretical curve can be used for interpreting many different subsurface structures.

A set of theoretical curves is shown in Figure 49 for a "two-layer" structure.

A two-layer structure means that a uniform surface layer of thickness D_1 and resistivity p_1 , overlies a large thickness of material with resistivity p_2 .

Interpretation is carried out as follows:

- Step 1: Plot the field data (apparent resistivity against electrode separation) on the same size double-logarithmic paper which has been used for the theoretical curves.
A different size of paper or a paper with a different number of logarithmic cycles may be used if the logarithmic cycles are identical in size on the theoretical and field sheets.
If it is desired to use paper with a different size of logarithmic cycle, then the curves must be replotted from the data of Roman (1960).
- Step 2: Superpose the sheet with the field data onto the sheet with the theoretical curves. It may be convenient to use a window or a light table.
- Step 3: Slide the top sheet over the bottom sheet until the field curve matches some one of the theoretical curves. Keep the axes of the graph paper parallel. Often, it will be possible to match only that portion of the field curve corresponding to the smaller electrode separations. This means that the subsurface structure includes more than the two layers, and that more complicated theoretical curves will be required. The two-layer interpretation will give useful information on the shallower subsurface materials, however.
- Step 4: On the graph of the field data, read the number along the vertical axis which overlies the horizontal heavy black line on the graph of theoretical data. This equals surface-layer resistivity, p_1 .
- Step 5: On the graph of the field data, read the number along the horizontal axis which overlies the vertical heavy black line on the graph of theoretical data. This gives the best estimate as to thickness of the surface layer, D_1 .

Step 6: Make a note of which theoretical curve provided the best match. Each curve is identified by a number which specifies the quantity,

$$\frac{(p_2 - p_1)}{(p_2 + p_1)}$$

Since p_1 is now known, p_2 may now be computed. This gives the resistivity of the material beneath the surface layer. Often, the field curve will fall between theoretical curves. If the field curve appears to be halfway between theoretical curves labelled 0.8 and 0.9, for example, we would take

$$\frac{(p_2 - p_1)}{(p_2 + p_1)} = 0.85$$

In practice, there are two difficulties with this method of interpretation by curve matching. The first is that the theoretical curves are computed for over-simplified subsurface structures. Uniform layers are assumed, separated by horizontal boundaries; the actual subsurface may be considerably more complex. In particular, any variations of material in a horizontal direction may cause serious deviations from the idealized case.

The second difficulty is that there are never enough theoretical curves to match all possible subsurface conditions. Curves or tables have been published for two layers (Roman, 1960), for three layers (Wetzel and McMurray, 1936), and for two, three and four layers (Mooney and Wetzel, 1956). It is usually necessary to estimate what curves for intermediate structures would look like. This results in considerable uncertainty in the interpretation.

A set of theoretical curves can be extremely valuable for qualitative understanding of the field data. By comparing one curve with another, the interpreter can visualize how the readings would be affected by, for example, greater thickness of one of the layers or lower resistivity of one of the layers. Considerable study is required, however, to achieve this intuitive feel for the expected behavior of resistivity readings.

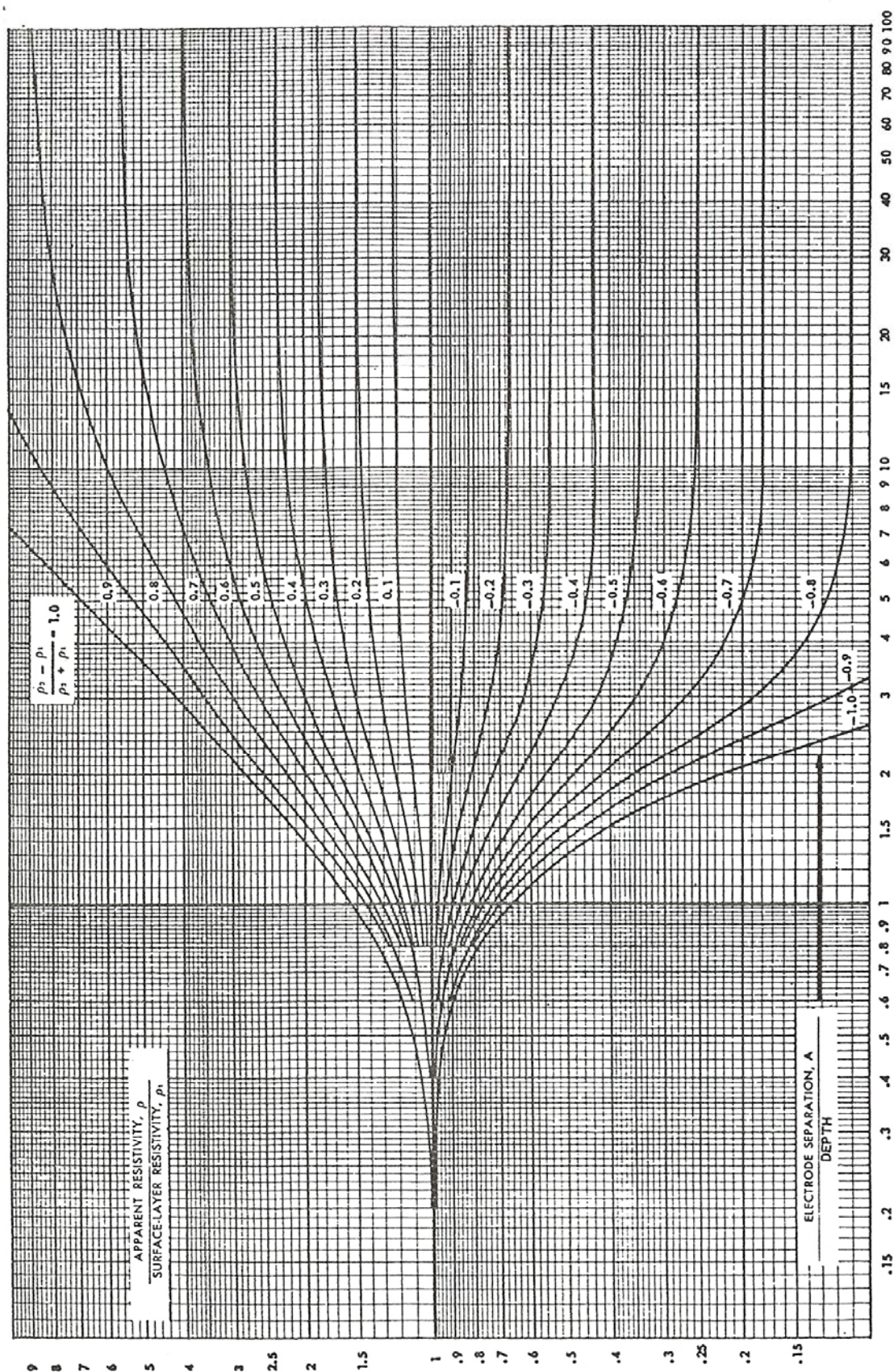
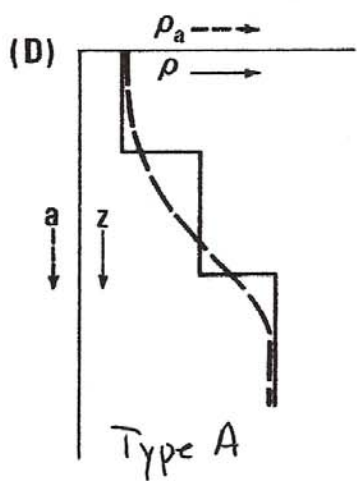
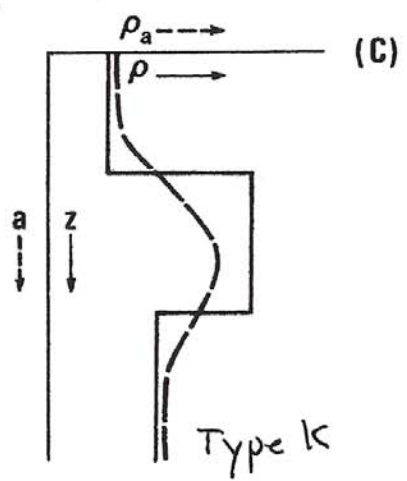
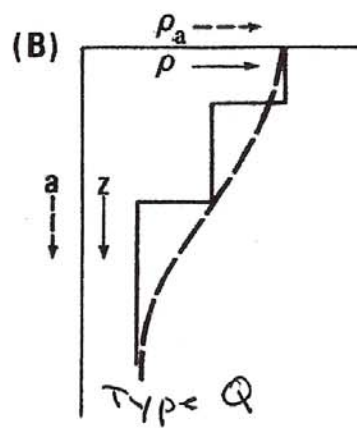
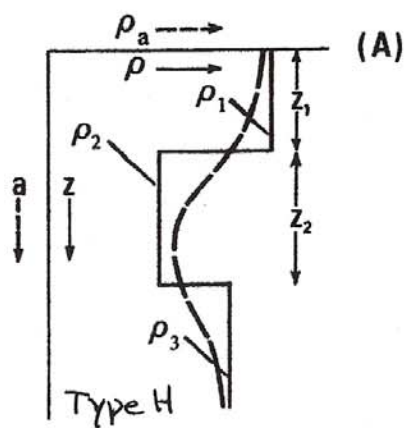
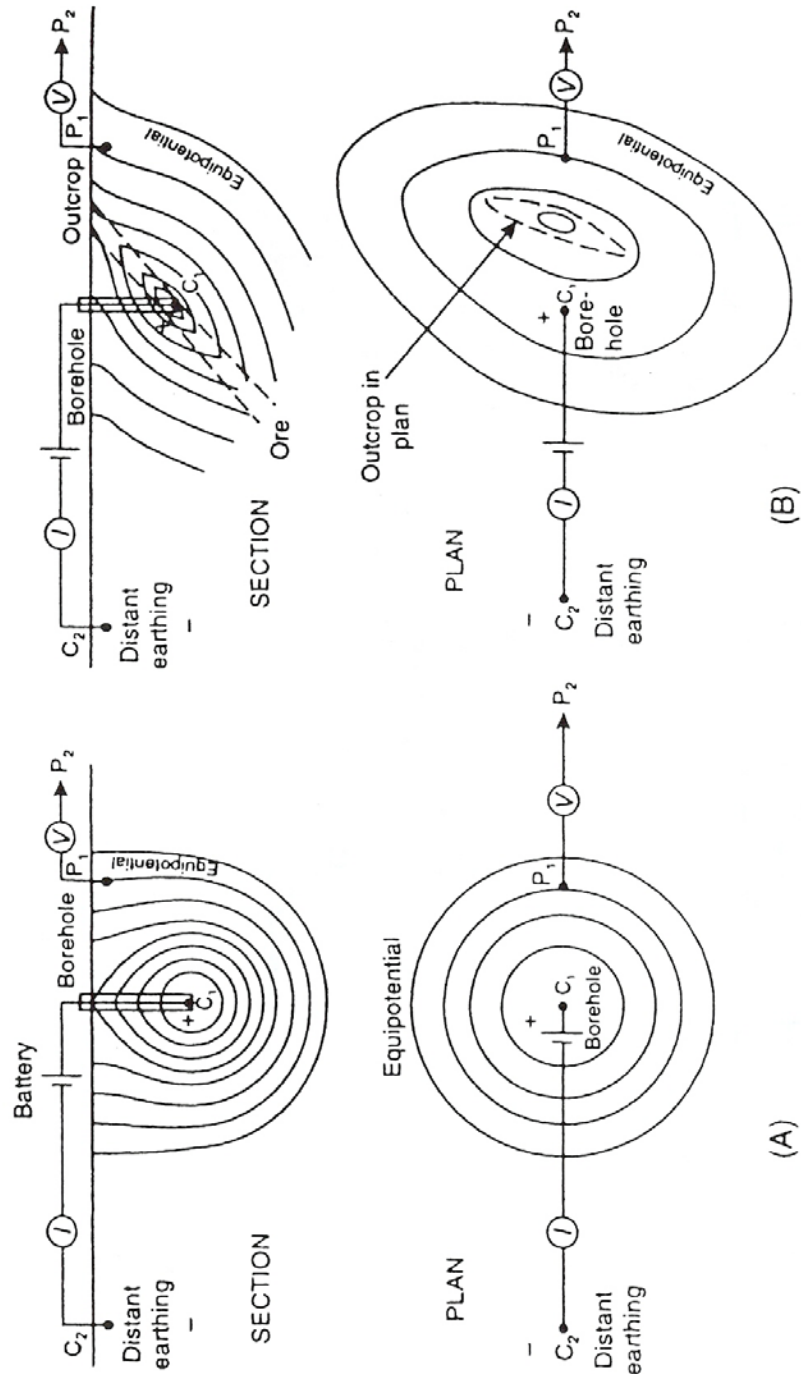


FIGURE 49: THEORETICAL TWO-LAYER RESISTIVITY SOUNDING CURVES





mise-a-la-messe