

Matched Filtering as a Separation Tool – review example

In any map of total magnetic intensity, various sources contribute components spread across the spatial frequency spectrum. A matched filter seeks to deconvolve the signal from one such source using parameters determined from the observations. In this example, we want to find an optimum filter which separates shallow and deep source layers. Matched filtering is a Fourier transform filtering method which uses model parameters determined from the natural log of the power spectrum in designing the appropriate filters.

Assumptions:

1. A statistical model consisting of an ensemble of vertical sided prisms (i.e. Spector and Grant, 1970; *Geophysics*, v. 35, p. 293-302).
2. The depth term $\exp(-2dk)$ is the dominate factor in the power spectrum: size and thickness terms for this example are ignored but they can have large impacts on the power spectrum. Depth estimates must be taken as qualitative.

For a simple example, consider a magnetic data set, with two convolved components, in the spatial frequency (wavenumber) domain:

$$\Delta T = T(r) + T(s)$$

where

$$T(r) = \text{regional component} = C * \exp(-k * D)$$

$$T(s) = \text{shallow component} = c * \exp(-k * d)$$

D = depth to deep component

d = depth to shallow component

k = radial wavenumbers ($2 * \pi / \text{wavelength}$)

c, C = constants including magnetization, unit conversions, field and magnetic directions.

Substitute from above, and

$$\Delta T = C * \exp(-k * D) + c * \exp(-k * d)$$

factor out the deep term, and

$$\Delta T = C * \exp(-k * D) * \left(1 + \frac{c}{C} * \exp(k * (D - d))\right).$$

We now have the expression for the total field anomaly in the spatial frequency domain as:

$$\Delta T = C \cdot \exp(-k \cdot D) * [1 + \frac{c}{C} * \exp(k \cdot (D-d))]$$

Note the term in brackets '[]'. If we multiply both sides of the equation by the inverse of the term in brackets, we have:

$$C \cdot \exp(-k \cdot D) = \Delta T * \frac{1}{1 + \frac{c}{C} * \exp(k \cdot (D-d))}$$

and we have separated, in the wavenumber domain, the signal from the deep layer.

Thus, the matched filter to separate the regional is:

$$\frac{1}{1 + \frac{c}{C} * \exp(k \cdot (D-d))}$$

We can separate the shallow layer with:

$$1 - \frac{1}{1 + \frac{c}{C} * \exp(k \cdot (D-d))}$$

The Fourier transformed total field anomaly, followed by its power spectrum, is:

$$\Delta T = C \cdot \exp(-k \cdot D) * [1 + \frac{c}{C} * \exp(k \cdot (D-d))]$$

$$\Delta T^2 = [C \cdot \exp(-k \cdot D) * (1 + \frac{c}{C} * \exp(k \cdot (D-d)))]^2,$$

and the filter is:

$$\frac{1}{1 + \frac{c}{C} * \exp(k \cdot (D-d))}$$

Now, we need to determine the unknown parameters (d, D, c, C) of the matched filter. Note that if we plot the natural log of the power spectrum versus wavenumbers that the exponential functions will be linear. This allows extraction of the necessary parameters for the matched filter from the radially averaged power spectrum as long as each of the layers shows up as a linear segment in the radially averaged power spectrum. In that case, the equation for the deep component is:

$$\text{Amp}^2 = C^2 \exp(-2Dk) \quad (\text{both sides are squared}).$$

$$\begin{aligned} \ln(\text{Amp}^2) &= \ln(C^2 \exp(-2Dk)) \\ &= \ln(C^2) - 2Dk \quad (\text{a linear equation in } \ln(\text{power})\text{-wavenumber space!}) \end{aligned}$$

Thus, the slope and intercept yield the depth and constant, respectively.

Plot $\ln(\text{amplitude}^2)$ vs wavenumber

- Pick deep linear segment

- Find the slope

$$\text{slope} = (6 - -6) / 0.60 = -20$$

$$\text{depth} = D = -\text{slope}/2 = 10$$

- Find C

intercept ~ 5.8

$$\ln(C^2) = 5.8$$

$$C = \sqrt{\exp(5.8)} = 18$$

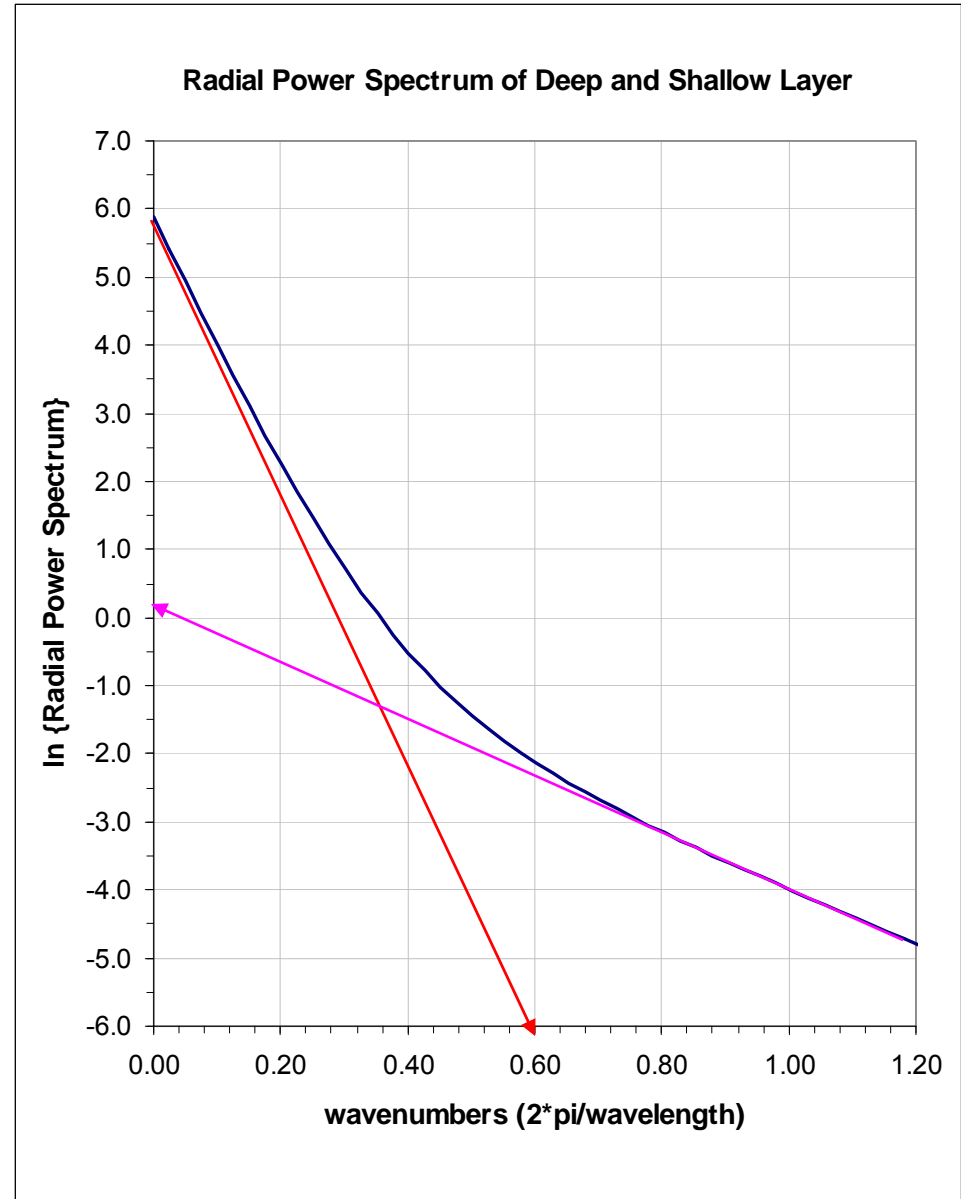
- Do the same for the shallow layer

depth = $d = 2$

$c = 1$

We now have the parameters (c, C, d, D) for the matched filter to separate the regional, deep layer:

$$\frac{1}{1 + \frac{c}{C} * \exp(k*(D-d))}$$



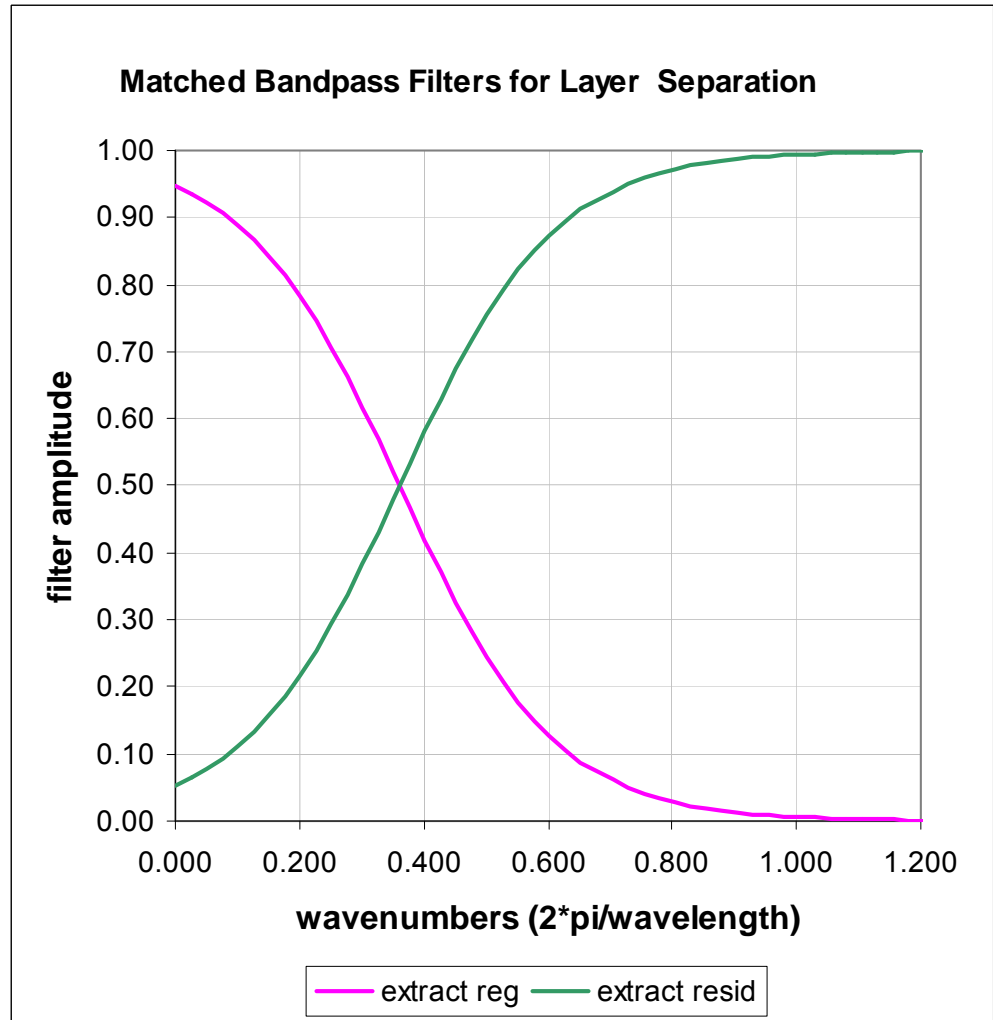
Graphing the matched filter:

$$\frac{1}{1 + \frac{c}{C} * \exp(k*(D-d))}$$

yields the magenta line and

$$1 - \frac{1}{1 + \frac{c}{C} * \exp(k*(D-d))}$$

yields the green line, the matched filter for the shallow layer. Multiplying (in 2D) these times the Fourier transform of the total magnetic field separates the layers subject to all assumptions.



Filter Choices for Matched Filtering with Oasis Montaj and the USGS Extensions

How do you choose a magnetic dipole layer versus magnetic half space when designing matched filters in the USGS Oasis Montaj extensions, and why?

First, go back to the basic linear filtering equation for calculating a forward magnetic anomaly from a randomly magnetized layer with vertical magnetization:

$$F(\Delta T(x,y)) = F(M(x,y)) * C_m * (\exp(-k*d) * (1 - \exp(-k*t))) \quad (\text{layer})$$

Now, let t increase to infinity to get a randomly magnetized half space

$$F(\Delta T(x,y)) = F(M(x,y)) * C_m * \exp(-k*d) \quad (\text{half space})$$

Here:

$F(\Delta T(x,y))$ = Fourier transform of the forward (modeled) result in 2D

$F(M)$ = Fourier transform of the random distribution of susceptibility in the layer

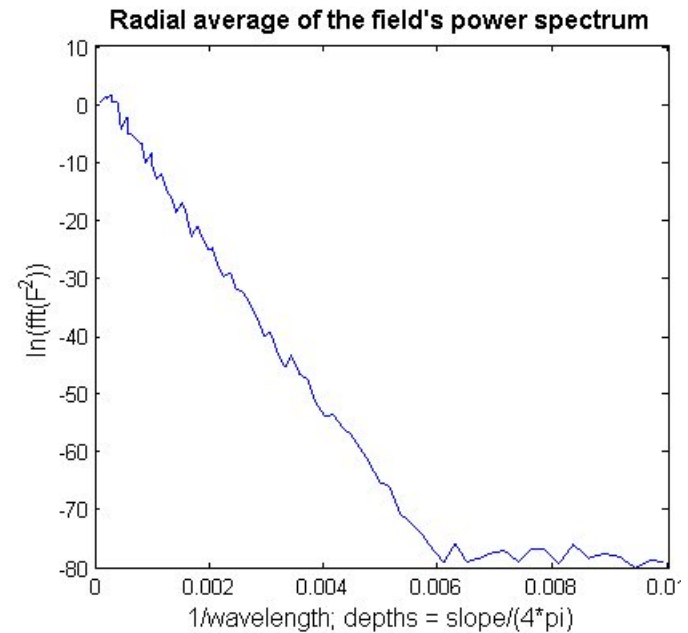
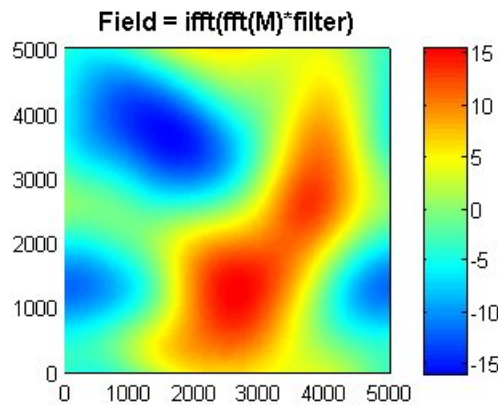
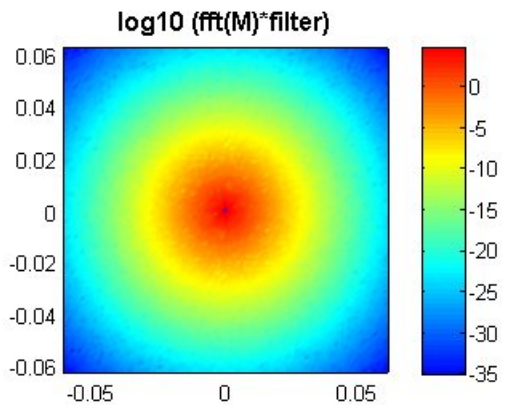
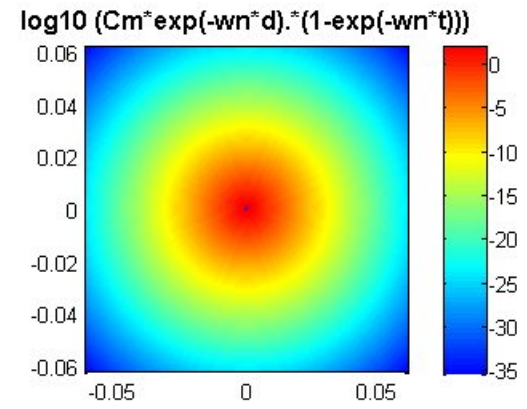
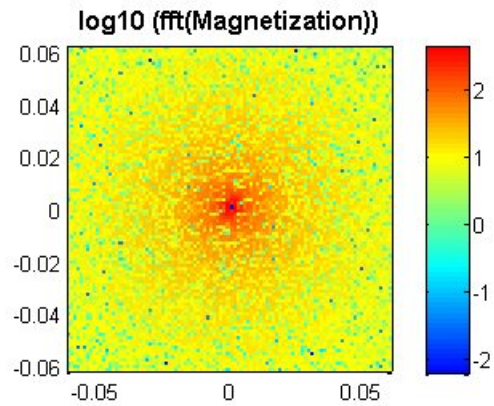
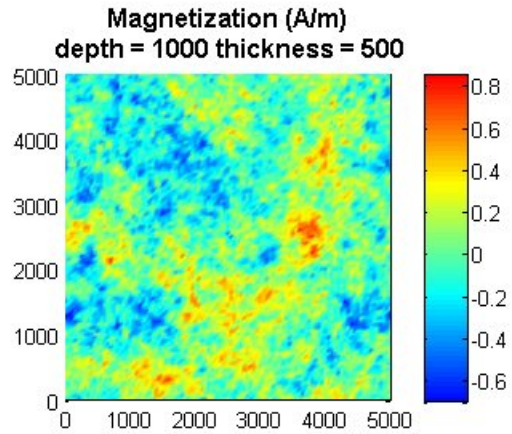
C_m = constant (units, magnetic directions)

$k = (k_x^2 + k_y^2)^{0.5}$, (k_x , k_y are wavenumbers ($2 * \pi$ / wavelength) in the x & y directions)

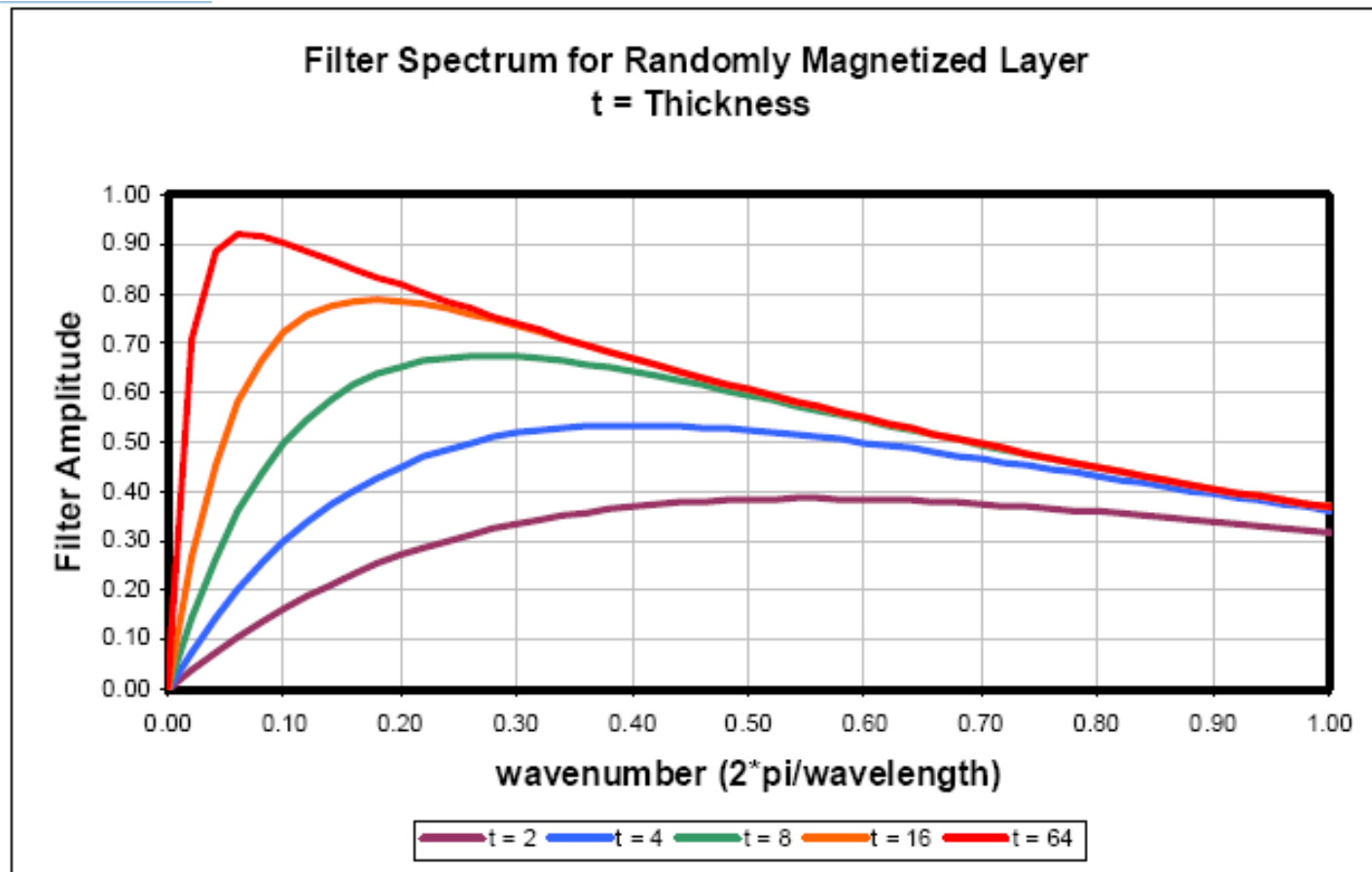
d = depth to the top of the layer

t = thickness of the layer.

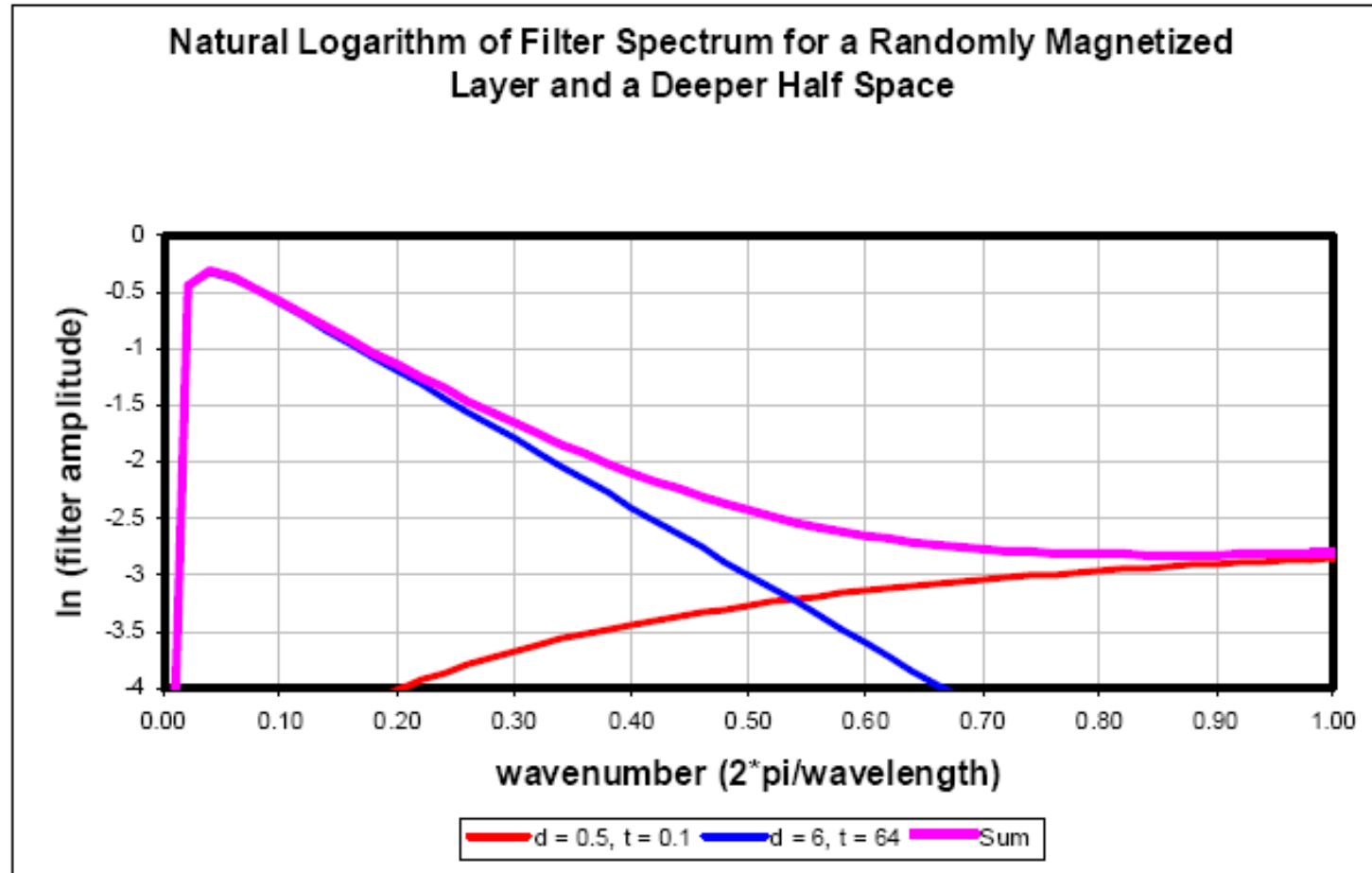
Calculating the total field anomaly from a randomly magnetized slab



In the above equation, when thickness (t) is close to depth (d), the natural logarithm of the radially averaged power spectrum of a thin magnetic dipole layer (code = 0) will be concave down at low wave numbers and linear at high wavenumbers. As the thickness increases, the concave down segment gets narrower and closer to the smallest wavenumbers. As thickness goes to infinity, and you are modeling a magnetic half space (code = 1), the natural log of the radially averaged power spectrum becomes linear at all wavenumbers.



Now, combine a thin shallow layer with a deeper half space:



So, when you select a layer versus half space you are fitting parameters to a different equation. A radially averaged spectrum produced by several equivalent source layers may have linear, concave up, and/or concave down segments.

Match Filtering Steps and Hints: Oasis Montaj

Select Grid Matched Filtering from the USGSV option of Oasis Montaj and select the ***Intialization*** entering your grid file and output file. The Design Filters begins by asking the user to select a layer code corresponding to a shallow equivalent source model for fitting the high wavenumber end of the spectrum. This choice is arbitrary.

Available codes and models are:

0 magnetic dipole layer

1 magnetic half-space or density layer

2 density half-space

Start by fitting the the right-hand side of the plot with the shallowest equivalent layer, usually a dipole layer (magnetics) or a density layer (gravity). Then move to the left for each subsequent equivalent layer.

Use thin-layer models for the shallowest layers; switch to half-space models at the depth where you want the greatest spectral separation.

You can try different models for the current layer. Remember to move on to the next layer when you are happy with the fit, and select "all done" after fitting the deepest (left-most) layer.

The non-linear least squares iterative improvement is performed until the RMS error is minimized. This step is recommended.

The spectrum of a single magnetic dipole layer (layer code 0) will appear linear at high wavenumbers but become concave downward and achieve a maximum at low wavenumbers.

The log averaged radial power spectrum of a single magnetic half-space or density layer (layer code 1) will appear linear at all wavenumbers. The spectrum of a single density halfspace (LAYER CODE 2) will appear linear at high wavenumbers but become concave upward at low wavenumbers. A spectrum produced by multiple equivalent source layers may exhibit combinations of linear, concave upward, and concave downward behavior. Theoretically code 0 should only be used on magnetic data and code 2 should only be used on gravity data. In practice the high wavenumber end of the spectrum, which corresponds to the shallowest equivalent source layer, can usually be fit to any of the three models. If layer code 0 or 2 are selected, the power spectrum will be corrected to the form of layer code 1 before plotting.

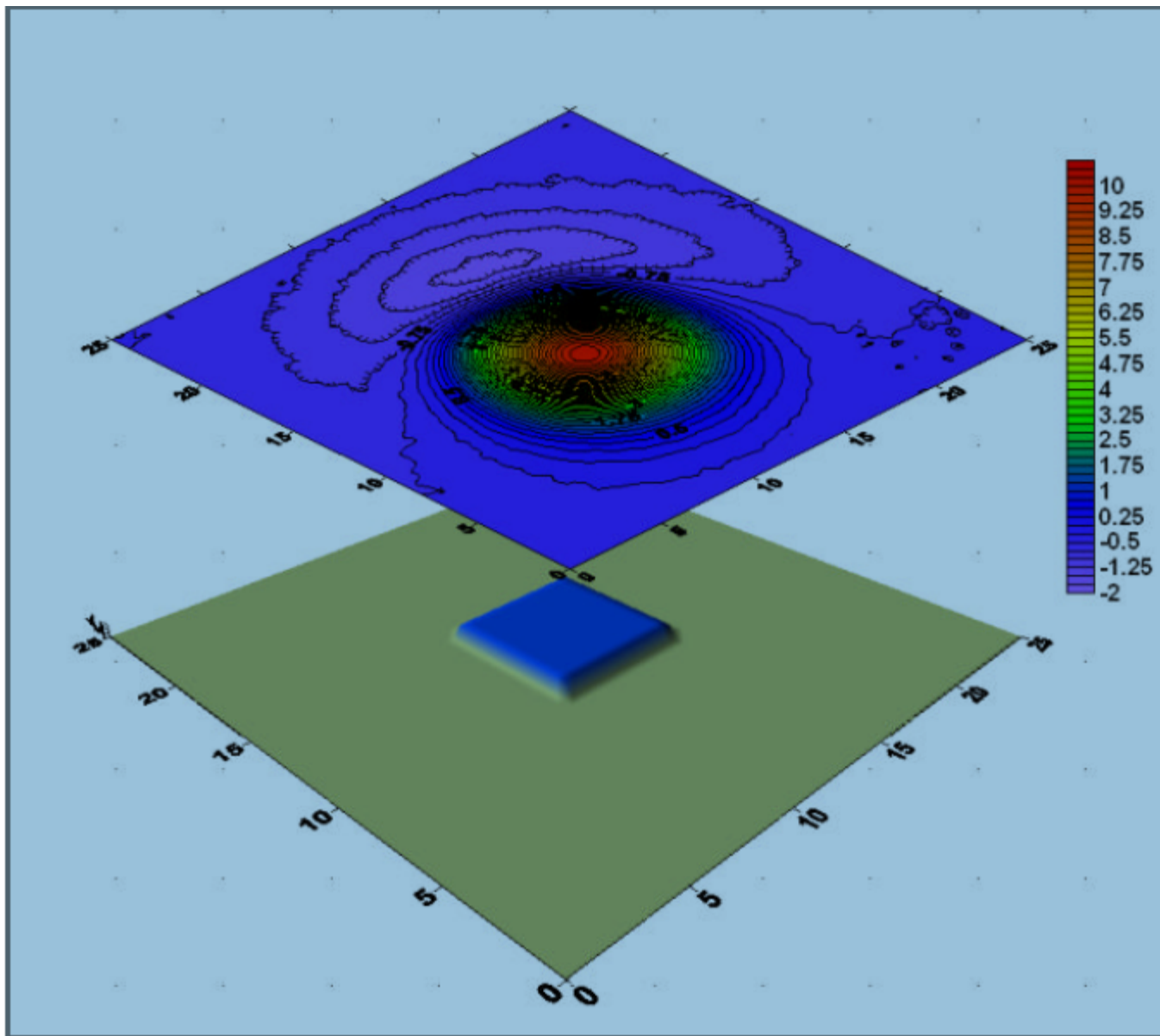
The program will next plot the (corrected) log averaged radial power spectrum and ask the user for the left intercept and bottom intercept of a line fitting the high wavenumber end of the curve. It may help to hold a straightedge up to the screen, and read the intercepts directly. If the line has no bottom intercept, enter the left intercept and zero; the program will then prompt for a right intercept. The indicated line will be drawn on the plot. A beep will indicate that the plot is finished and the user must press the enter key.

The user will next be given the option of trying again using the same or a different model (layer codes 0, 1 or 2), or accepting the fit for this high wavenumber end of the spectrum and proceeding to model the adjacent lower wavenumber range of the spectrum using a deeper equivalent source layer ('next layer'). If next layer is selected, the effects of the modeled equivalent source layer will be removed from the spectrum, and the residual spectrum will be plotted.

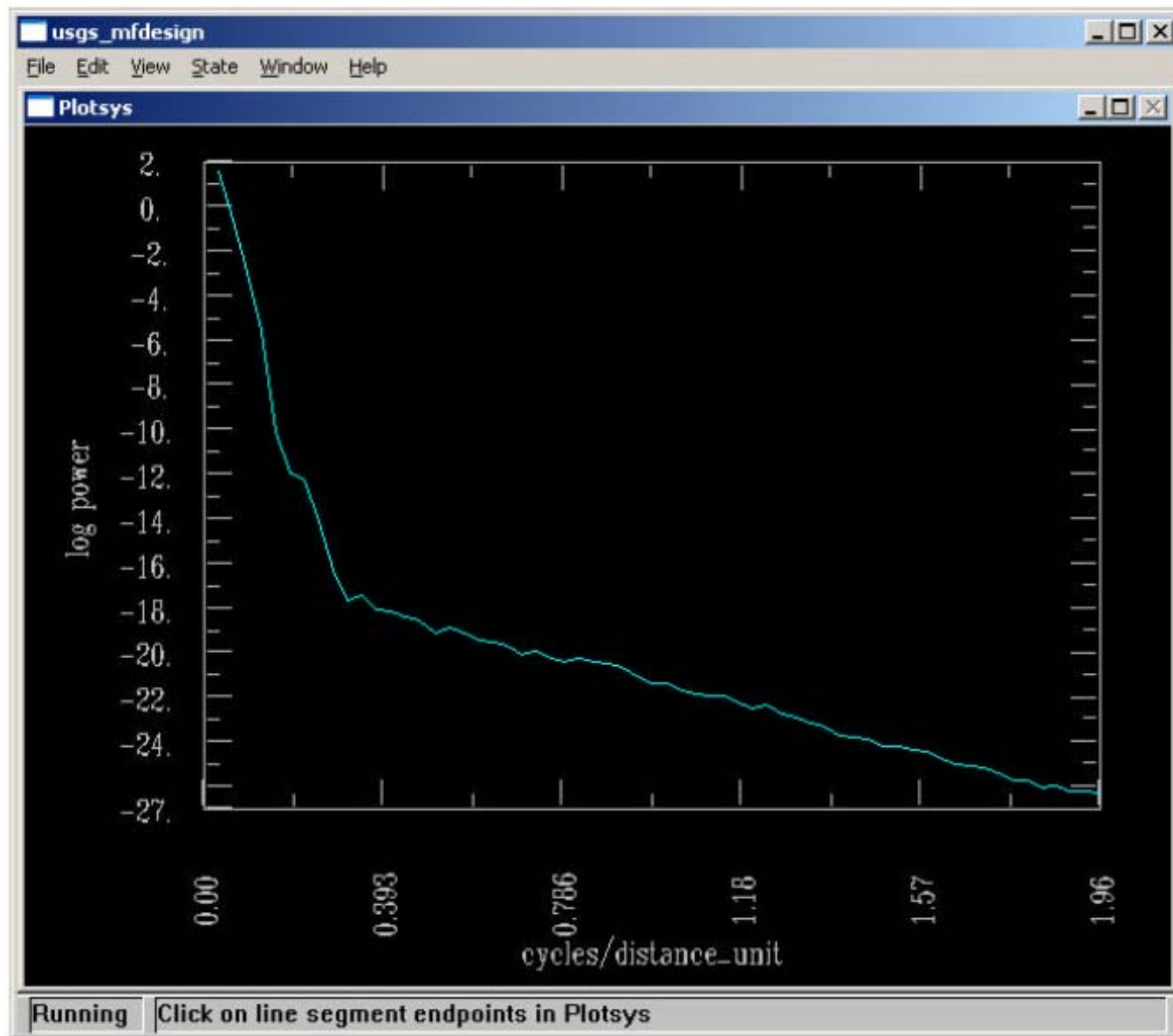
The program will next ask the user for a layer code corresponding to the equivalent source model to be used to fit the next lowest part of the spectrum. Proceed as before by selecting a layer code between 0 and 2 and fitting a line to this part of the spectrum. When the fit is acceptable to the next deepest layer.

When the user has fit an acceptable line to the lowest wavenumber part of the spectrum, the model is completed by clicking the appropriate radio button. The observed and modeled spectra are then plotted, and the user is asked if iterative improvement of the model is desired. Iterative improvement can be performed a step at a time (step-by-step option) or recursively until the percent improvement becomes zero or negative (forever option). The improved model spectrum will be plotted and the model parameters listed. The user will be given a second chance to iterate on the model, and a chance to start completely over.

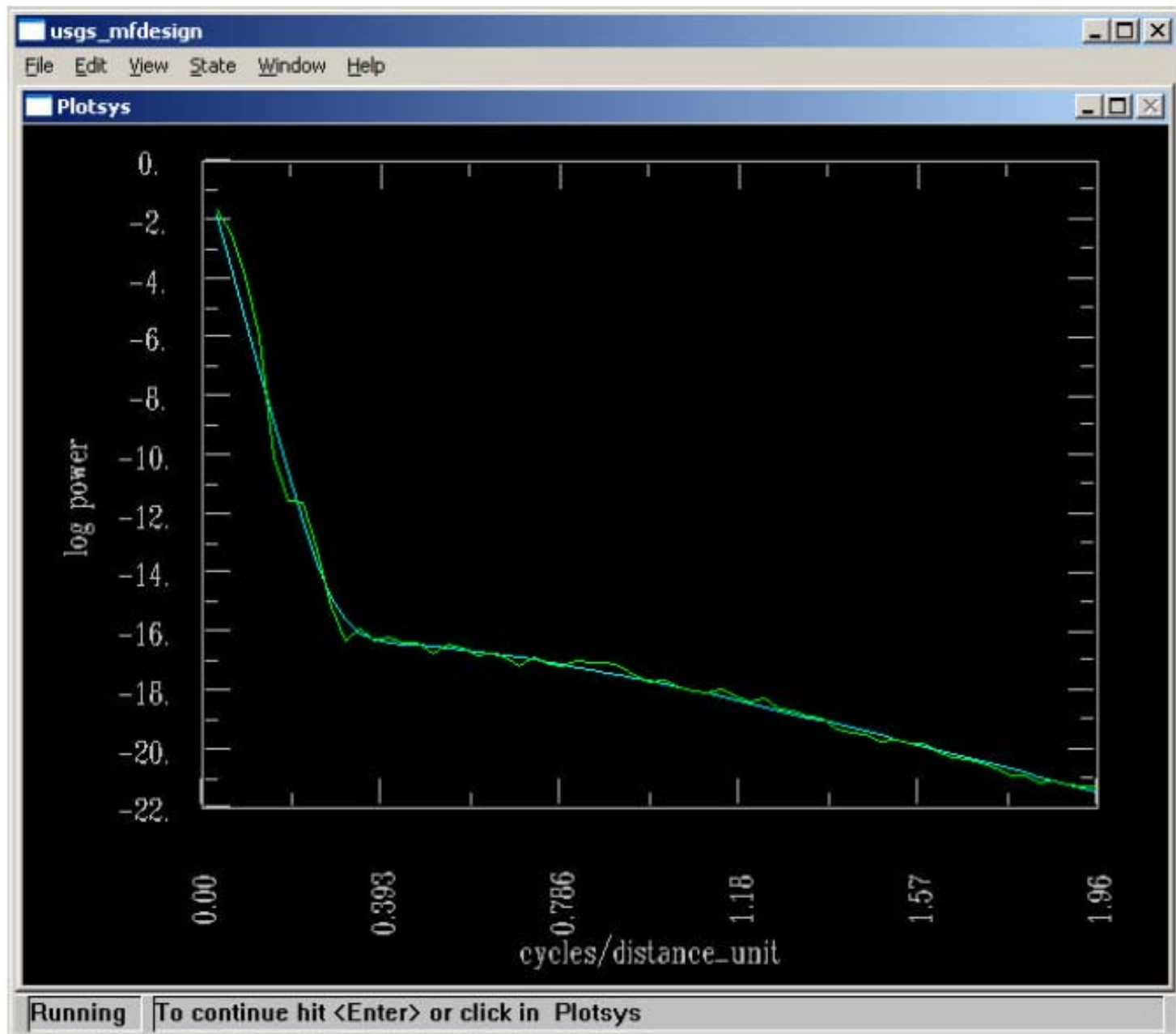
Once the model has been accepted, the program will plot the bandpass filters corresponding to the equivalent source layers, and list the crossover wavelengths of the filters using the option ***Apply Filters***. After ***Design Filters*** has completed, the current, observed and model spectra and the current bandpass filters can be redisplayed by running using the program ***Show Filters***.



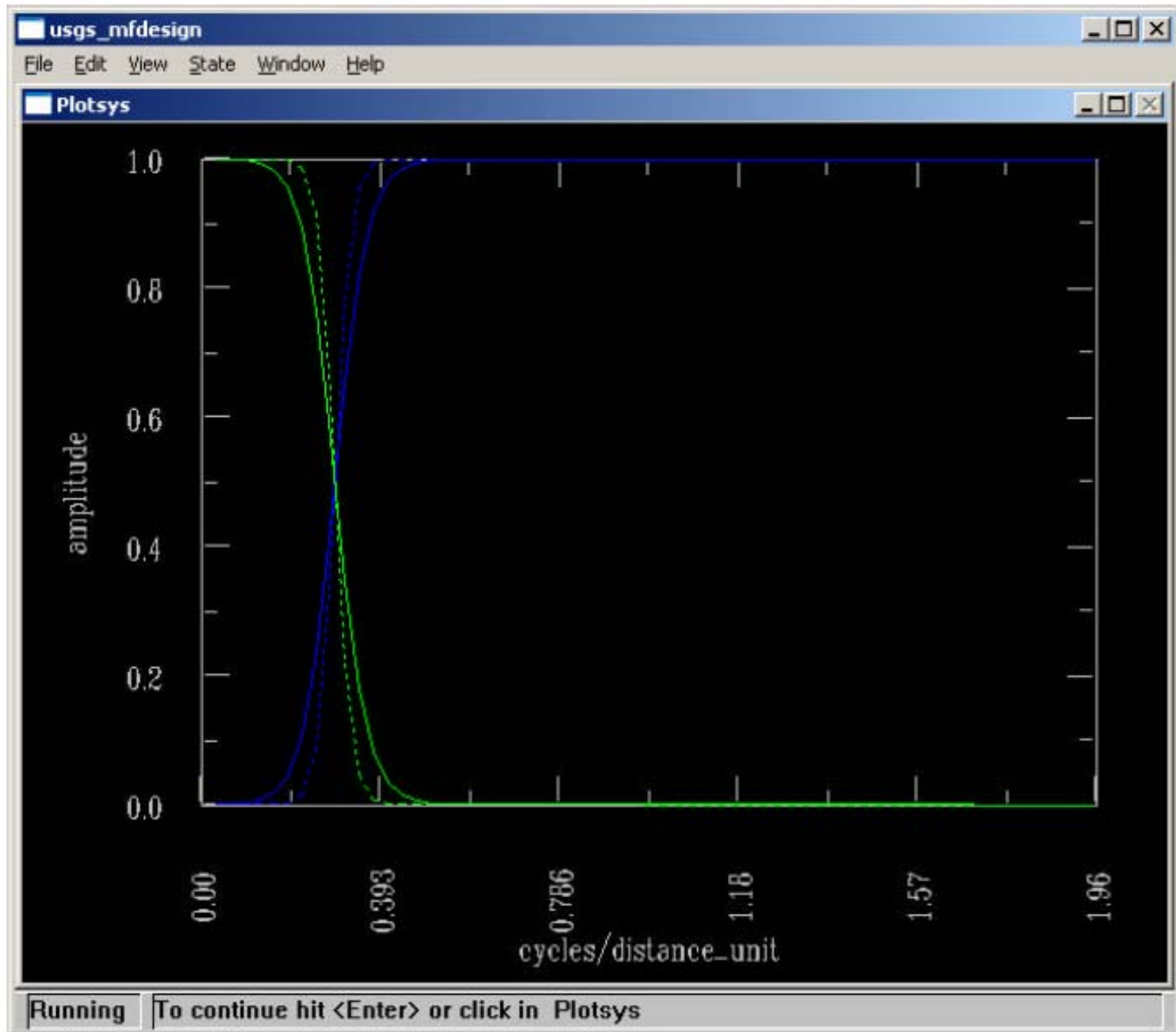
For the demo data, I modeled a vertical prism 4 kilometers below the surface and added a few percent noise to the resulting total field anomaly. The added noise is in the observations and represents observational error. I did that by using Surfer's Grid Node Editor to turn the grid to a .dat file. I then added a column of noise (use data transform and $RANDn(\text{average}, \text{StDev})$) and added that to the observation column. I then regridded the data.



This is the radially averaged power spectrum for the noisy data above. Note that there are two distinct linear segments, representing shallow and deep equivalent sources. The low slope segment runs from a wavelength of about 2.5 kilometers to 0.5 kilometers.



This is MFDESIGN's fit to the power spectrum after iterative adjustment.

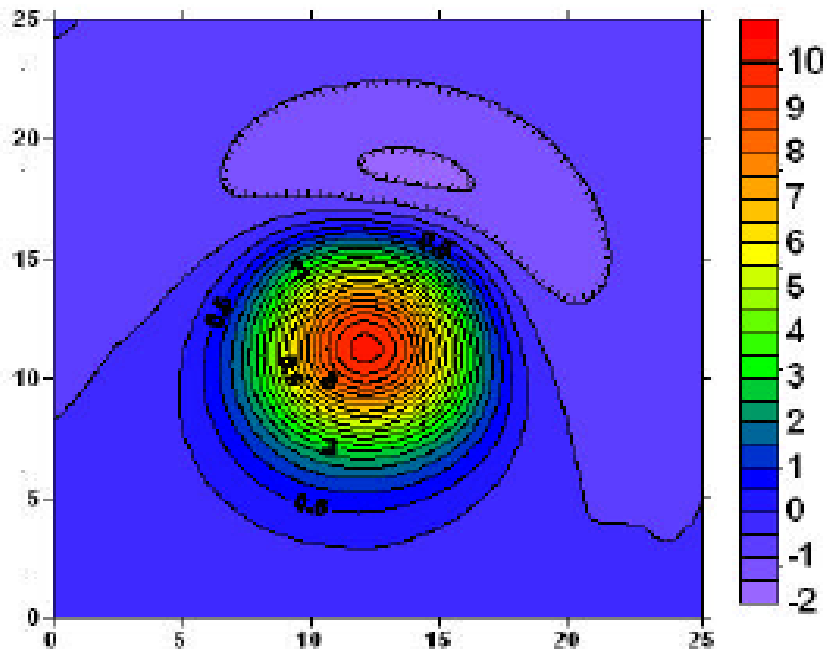


These are the low pass and high pass filters that represent the two linear segments in the radially average power spectrum

The screenshot shows a software window titled "usgs_mfdesign" with a menu bar containing "File", "Edit", "View", "State", "Window", and "Help". Below the menu bar is a sub-window titled "Plotsys" with a black background and white text. The text in the "Plotsys" window reads: "Solid lines are amplitude filters; dashed lines are Wiener filters." followed by a table of filter parameters.

Bandpass Wavelengths:	Depth	Wmin	Wmax
	0.4182	0.5091	3.507
	4.432	3.507	0.000

This shows the depth of the equivalent layers isolated in the power spectrum



The upper map is the anomalous field from the long wavelength (low wavenumber) part of the spectrum. The lower map is from the high wavenumber (short wavelength) part of the spectrum. I generated the noise as a random function (RANDn(average, StDev) in a Surfer worksheet and added it to the original data. The lower map is a good characterization of random noise, the upper map an excellent representation of the field without the noise.

