GEOPHYSICAL RESEARCH LETTERS, VOL. 32, L16309, doi:10.1029/2005GL023594, 2005

Probabilistic analysis of rockfall frequencies during an andesite lava dome eruption: The Soufrière Hills Volcano, Montserrat

E. S. Calder,[1] J. A. Cortés,[2] J. L. Palma,[1] and R. Luckett[3]

Received 23 May 2005; revised 18 July 2005; accepted 1 August 2005; published 25 August 2005.

[1] A probabilistic analysis of rockfalls has been undertaken using seismometer-determined information on the timing and duration of rockfalls from the Soufrière Hills lava dome, Montserrat, between 1995 and 1997. Repose intervals between rockfalls fit log logistic survivor distributions. Differences in the log logistic survivor function for different sub-sets of data are accommodated by variation of the parameter k, which varies from 0.7 to 1.6, and sample median, μ. When extrusion rate is assumed to be ≈ 0 m^3/s, k is ≤ 1 denoting that the hazard function has a continuously decreasing form, contrasting to the hazard maximum present during periods of extrusion. This analysis illustrates a direct relationship between k, μ, and lava extrusion rate. Extrusion rate becomes increasingly significant in determining rockfall frequencies when rates are relatively high, whereas other factors become increasingly significant when rates are low.

Citation: Calder, E. S., J. A. Cortés, J. L. Palma, and R. Luckett (2005), Probabilistic analysis of rockfall frequencies during an andesite lava dome eruption: The Soufrière Hills Volcano, Montserrat, *Geophys. Res. Lett.*, *32*, L16309, doi:10.1029/2005GL023594.

1. Introduction

[2] Rockfalls occur along high-angle failure planes on the outer, largely degassed, carapace of growing lava domes and involve discrete blocks that roll, bounce or slide downhill. The intensity of rockfall activity generated at dome margins and the propensity for domes to collapse can be related to magma extrusion rate. Although somewhat oversimplified and often based on observational evidence or alluded to indirectly, this is indeed an intuitive relationship. The eruption of the Soufrière Hills lava dome, Montserrat, has now continued for 10 years, over which time there have been a number of references made to this relationship: Rockfall activity correlates with directional and temporal changes in lobe extrusion [*Watts et al.*, 2001] as well as changes in eruption style [*Calder et al.*, 2002]. Increased rockfall activity occurred as an immediate result of new batches of magma intruded into the base of the dome [*Voight et al.*, 1999]. *Luckett et al.* [2002] illustrated extrusion rate-dependent variations in the frequency content of rockfall seismic signals linked to increased gas resonance during periods of rapid extrusion. Extending this relationship to larger, pyroclastic flow-generating collapses, first order links have found between extrusion rate and collapse volumes [*Calder et al.*, 2002; *Simmons et al.*, 2005] and at Unzen, daily numbers of block-and-ash flows roughly correlated with dome growth rate [*Ui et al.*, 1999].

[3] This paper quantifies the relationship between rockfall intensity and extrusion rate through a probability analysis approach. We investigate the variations in the frequency of occurrence of rockfalls at the Soufrière Hills lava dome between 12 December 1995 and 6 September 1997. Information on the timing and duration of rockfalls was collected by the short period seismic network at the Montserrat Volcano Observatory (MVO). These data comprise over 14,000 rockfall events, which, combined with patterns of dome growth and extrusion rates [*Sparks et al.*, 1998; *Watts et al.*, 2001], provide a unique data set. If rate determined variations occur in the patterns of rockfall timings, automatically collected data on rockfalls could be used, in the absence of direct measurements, as a proxy for extrusion rate.

2. Rockfall Data

[4] For the period November 1995–September 1997, the duration of seismic signals produced by rockfalls were measured by MVO staff manually from the paper records of the short-period seismometers. The duration of the signal produced by the avalanche, corresponds to the time the rockfall spent travelling over the ground surface (distance covered and velocity) and dispersion effects of the seismic signal. In addition, long signal durations can be produced by sequences of rockfalls excavating into the dome for sustained periods. Repose intervals between rockfalls, are herein defined, as the time elapsed between the onsets of the signals when they trigger the seismic network. The duration frequency distribution shows an asymmetric distribution with a mode of 40–50 s. durations (Figure 1a). The repose frequency distribution shows a continuously decreasing form (Figure 1b), reflecting the tendency that most rockfalls occur within rapid successions (sample mode occurs within the 0–10 min. bin).

[5] To assess rate-determined variation we use extrusion rates estimated from direct measurements of dome volume [*Sparks et al.*, 1998]. Six subsets of rockfall data were separated, which were associated with periods of relatively well constrained extrusion rates and where style of dome growth is well documented [*Watts et al.*, 2001] (Table 1). (i) 1–30 April 1996 was characterised by the repeated extrusion of large vertical spines with low extrusion rates. (ii) 18–30 September 1996 immediate after the sub-Plinian explosion of 17 September, when no extrusion was taking place and before a new dome appeared in the crater. (iii) 16 December 1996 to 2 January 1997 was charac-

---

[1] Department of Earth Sciences, Open University, Milton Keynes, UK.
[2] School of Earth and the Environment, University of Leeds, Leeds, UK.
[3] Montserrat Volcano Observatory, Montserrat, West Indies.

0094-8276/05/2005GL023594$05.00

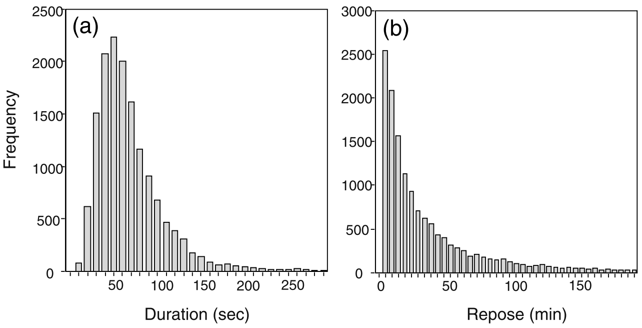

Figure 1. Rockfall data from the period 12 December 1995 to 6 September 1997, a) Duration frequency distribution, in 10-second bins and b) Repose interval frequency distribution, in 10-minute bins.

terised by the formation of a megaspine and pancake lobe on the dome summit with rapid extrusion rates. (iv) 1–28 February 1997 was characterised by moderate extrusion rates. (v) 17 May–22 June 1997 represents the build-up to the major 25 June collapse, when growth was occurring on the northern flanks and extrusion rates were elevated. (vi) 4–12 August 1997 was characterised by a succession of 13 vulcanian explosions and inferred high extrusion rates. Here we assume that all lava extrusion was accommodated by explosive activity and there was no net intrusion into the dome itself (thus, $0\ m^3/s$).

[6] Plots of repose time against rockfall duration for these sub-sets commonly show a scatter of data with a, negatively trending, maximum limit. For the 5 large samples we define a linear maximum ξ_{95}, under which >95% of the data occur, by calculating the slope and intercept of a least squares regressions through the data with the maximum repose time in each 5-second duration bin (Figure 2a). For ξ_{95} a range of gradients from -0.06 to 0.4 is found (Figure 2b and Table 1), where steeper gradients are associated with higher extrusion rates. These data illustrate that when extrusion rate is low, rockfalls have shorter durations and longer repose periods while those of higher extrusion rates show a shift toward longer durations with shorter repose intervals.

3. Probability Analysis

[7] Rockfall repose intervals have been analysed using failure analysis [*Cox and Oakes*, 1984]. If a component is put under stress at time, $t = 0$, and observed until it fails (in this case when material is shed from the dome), the time to failure, T, may be considered as a continuous random variable with some probability density function. Thus, the survival (or non-failure) of a system at time t is defined as $S(t) = P\ (T > t)$ in which S(t) is the survivor function [*Cox and Oakes*, 1984]. The survivor function values for rockfall data consistently fit log logistic distributions (Figure 3) at the 95% confidence level using quadratic error minimisation. The survival function of the log logistic density distribution is given by (1), in which the expected value μ is estimated using the median repose and the parameter k is adjusted for best fit.

$$S(t) = \frac{1}{1 + \left(\frac{t}{\mu}\right)^k} \tag{1}$$

Differences in the log logistic survivor function for each sub-set of data are accommodated by variation of μ and the parameter k, which varies from 0.7 to 1.6 (Figure 3b and Table 1). As k increases (Figure 3a, inset), the survival function steepens, reflecting a narrowing of the repose interval cluster. Variation in μ produces a horizontal shift of the curve on the x-axis. The relationship with extrusion rate is linear for μ and k (Figures 4a and 4b). The (instantaneous) failure rate, or hazard function h(t), represents the probability that the system will fail during the next $t + \Delta t$ time units, given that the system has still not failed at time t and is given by;

$$h(t) = \frac{\frac{dS(t)}{dt}}{S(t)} \tag{2}$$

$\Delta t \times h(t)$ represents the proportion of collapses that will occur between t and $t + \Delta t$. For a log logistic distribution, the hazard function has the expression;

$$h(t) = \frac{k t^{k-1} \left(\frac{1}{\mu}\right)^k}{1 + \left(\frac{t}{\mu}\right)^k} \tag{3}$$

When $0 < k \leq 1$, hazard decreases with time (Figure 4b, inset), while values of $k > 1$ denote that the hazard function, has a maximum at time t*, given by;

$$t^* = \mu(k-1)^{\frac{1}{k}} \tag{4}$$

The survivor function of both periods of no dome growth (September 1996 and August 1997) have respective k

Table 1. Summary Data

Period	Extrusion Rate,[a] $m^3 s^{-1}$	Events, n	Repose Median M, min	Repose Mean, min	Slope of ξ_{95}	Intercept	k	r^2	Std. Error[b]	t*, min
1 Apr – 30 Apr 1996	1.7	335	46	116.5	−0.11	202.37	1.1	0.997	0.007	5.67
18 Sep – 30 Sep 1996	0	137	24	125.83	−0.06	124.77	0.7	0.993	0.01	n/a
16 Dec – 2 Jan 1997	5.1	393	32	65.94	−0.40	311.34	1.2	0.997	0.008	8.37
1 Feb – 28 Feb 1997	2.9	519	37	77.52	−0.16	136.99	1.1	0.996	0.011	4.56
17 May – 22 June 1997	5.7	1085	29	47.73	−0.26	195.14	1.6	0.997	0.01	21.07
4 Aug – 12 Aug 1997	0	59	74	184.80	n/a	n/a	1.0	0.982	0.02	n/a

[a] Based on the 7-day running average [*Sparks et al.*, 1998] with no growth assumed for Sep. 1996 and Aug. 1997.
[b] Related to the probability obtained using log logistic function with respective k and μ values.

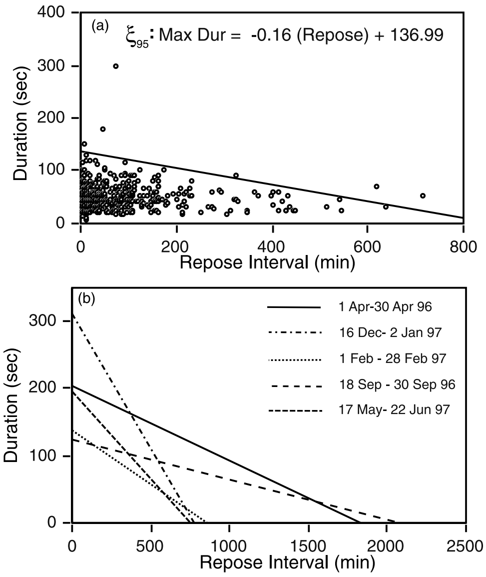

Figure 2. Duration versus repose time for rockfalls that occurred during a) the period 1–28 February 1997, with regression line calculated to estimate position of ξ_{95}. b) The calculated ξ_{95} maximum for the 5 main periods. Extrusion rates, slope gradients and intercepts are listed in Table 1.

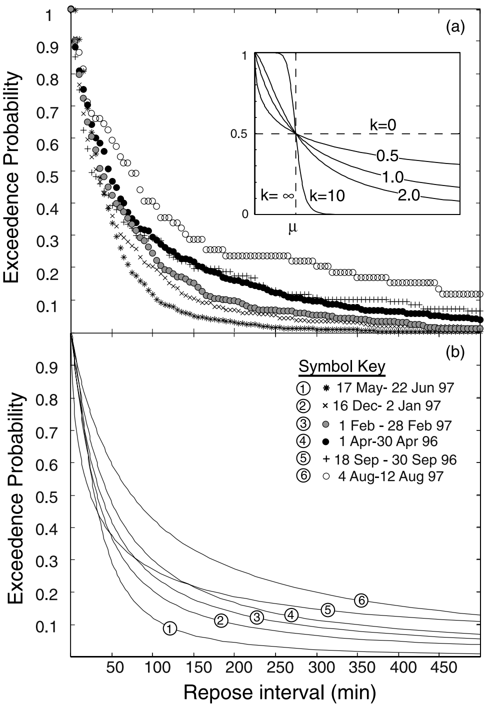

Figure 3. a) Survival function data for the 6 periods listed in Table 1. Inset illustrates the effects of varying k on the shape of the probability functions. A change in μ, produces a horizontal shift in the position of the curve along the x-axis. b) Best-fit log logistic survival function curves calculated with 95% confidence level for the data in a). Note, the August 1997 data has a lower r^2 value and standard error a factor of 2 larger than the other samples due to smaller sample size (Table 1).

values of 0.7 and 1 indicating that the potential hazard decreases with time (Table 1). So although rockfalls occurring during dome growth and no-growth both have similar survival functions, their characteristic repose frequencies can, on the basis of this analysis, be distinguished by the form of their hazard functions. For the $k > 1$ values, the time period at which one is most at risk from the given failure taking place for each of the periods is given in Table 1.

4. Discussion

[8] During 1995–1997, dome growth at the Soufrière Hills volcano largely occurred by extrusion of lobes or spines with rates in the order 1.5–7 m^3/s. ξ_{95} illustrates that the changing rate of growth of the lava dome is reflected in both the frequency of occurrence of rockfalls and the duration of their seismic signals, which in the absence of other systematically measurable parameters can loosely be equated to rockfall magnitude. More material is shed more frequently, when extrusion rates are high. This relationship is reinforced by Figure 4a, where the median repose interval is shown to be rate-dependent.

[9] Rockfalls, like vulcanian explosions [*Connor et al.*, 2003], follow log logistic distributions rather than alternative failure laws such as exponential distributions or Weibull distributions. Physical processes invoked for instigating failure, both during growth and periods of intermission,

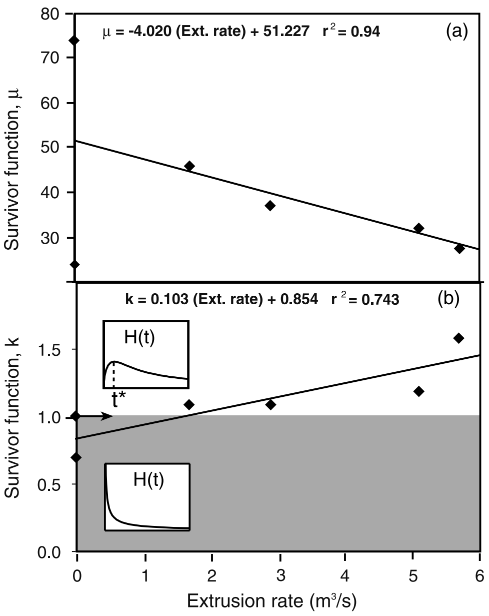

Figure 4. a) A linear relationship is found for extrusion rate versus sample median, μ when extrusion rate is non-zero. b) Extrusion rate versus survivor function k. Note, increasing the extrusion rate for the August 1997 data from 0 m^3/s to 1 or 2 m^3/s, which may be a more realistic value for this period if not all extrusion was being accommodated by the explosive activity, would improve the regression r^2 value to 0.80 and 0.82 respectively. The inset shows the decreasing hazard function H (t), when $0 < k \leq 1$ (shaded field), and a hazard maximum at time t* after the last event when $k > 1$ (white field).

have, therefore, to account for this distribution. A caveat being, intrinsic in the survival analysis, is the assumption that the events are considered independent, which is not necessarily always the case with rockfalls. The significance of parameter k, related to the inverse of the sample variance, is critical (Figure 3a, inset): distributions, with high variations in repose periods generate low k-values. Events with repose intervals that are narrowly spread around the median, by whatever the driving process, will have higher k-values. In the extreme case, this tends towards the events becoming periodically distributed in time. Furthermore, for predictions based on the log logistic distribution, the interval of confidence is narrower, and thus the reliability of the prediction is higher, when k is higher. The k-value of 4 obtained for the vulcanian explosions at Montserrat reflects the control exerted by gas pressurisation cycles and the development of permeability [*Connor et al.*, 2003]. Rockfalls have low k-values, <2, indicative of a less systematic process. The slight increase of parameter k with extrusion rate seen in the rockfall data (Figure 4b, r^2 of 0.74–0.82) reflects a subtle modification of the scattering by a process tied to lava extrusion. This is also intrinsic in Figure 2b, where the spread of repose data decreases with increasing extrusion rate, concentrating the data to the left and increasing the gradient of $\xi_{95.}$ During the May–June 1997 period, when cyclic pulsations in lava extrusion occurred [*Voight et al.*, 1999], higher k-values could be related to periodicity of the rockfalls which were common during the deflatory part of each cycle.

[10] It follows that in the modelling of failure mechanisms, extrusion rate becomes more relevant as a parameter when it is higher, whereas other factors may become increasingly significant when rates are low. Small-scale instabilities on lava domes, generating rockfalls occur as a result of a number of different physical processes. However, thrust forces associated with active lava lobe intrusion or extrusion [*Calder et al.*, 2002; *Watts et al.*, 2001] and oversteepening and the propagation of thermal fractures are directly related to extrusion rate. As are those processes responsible for deeper-seated failures; gas overpressurization [*Voight and Elsworth*, 2000] and heterogeneous strength distribution [*Simmons et al.*, 2005]. We infer that it is the results of these physical processes occurring that will most influence the rockfall frequency distribution when extrusion rates are high. Factors that play a role in dome stability, which are independent of extrusion rate include destabilization by seismic acceleration during earthquake swarms [*Calder et al.*, 2002] and external forcing such as rainfall-induced overpressurization or talus apron erosion [*Matthews et al.*, 2002; *Simmons et al.*, 2004; *Elsworth et al.*, 2004]. These mechanisms are considered to have an increasing importance in determining rockfall frequencies when extrusion rates are low. Finally, rockfalls which occur during periods of growth intermission, can on the basis of this analysis, be distinguished by the form of their hazard functions. Together these results deliver an alternative, testable, means of estimating extrusion rate based on analysis of automatically collected seismic data when direct measurements of dome volume cannot be made.

[11] **Acknowledgments.** The authors gratefully acknowledge RSJ. Sparks and G. Wadge for helpful reviews. This work was supported by a Royal Society Dorothy Hodgkin Fellowship to ESC and published with permission of BGS.

References

Calder, E. S., et al. (2002), Mechanisms of lava dome instability and generation of rockfalls and pyroclastic flows at Soufrière Hills Volcano, Montserrat, in *The Eruption of Soufrière Hills Volcano, Montserrat, From 1995 to 1999*, edited by T. H. Druitt and B. P. Kokelaar, *Geol. Soc. London Mem.*, *21*, 483–516.

Connor, C. B., et al. (2003), Exploring links between physical and probabilistic models of volcanic eruptions: The Soufrière Hills Volcano, Montserrat, *Geophys. Res. Lett.*, *30*(13), 1701, doi:10.1029/2003GL017384.

Cox, D. R., and D. Oakes (1984), *Analysis of Survival Data*, Chapman and Hall, London, 201 pp. 13–31.

Elsworth, D., et al. (2004), Thermal-hydrologic mechanism for rainfall-triggered collapse of lava domes, *Geology*, *32*(11), 969–972.

Luckett, R., B. Baptie, and J. Neuberg (2002), The relationship between degassing and rockfall signals at Soufrière Hills volcano, Montserrat, in *The Eruption of Soufrière Hills Volcano, Montserrat, From 1995 to 1999*, edited by T. H. Druitt and B. P. Kokelaar, *Geol. Soc. London Mem.*, *21*, 595–602.

Matthews, A. J., et al. (2002), Rainfall-induced volcanic activity on Montserrat, *Geophys. Res. Lett.*, *29*(13), 1644, doi:10.1029/2002GL014863.

Simmons, J., D. Elsworth, and B. Voight (2004), Instability of exogenous lava lobes during intense rainfall, *Bull. Volcanol.*, *66*, 725–734.

Simmons, J., D. Elsworth, and B. Voight (2005), Classification and idealized limit-equilibrium analyses of dome collapse at Soufrière Hills volcano, Montserrat, during growth of the first lava dome: November 1995–March 1998, *J. Volcanol. Geotherm. Res.*, *139*, 241–258.

Sparks, R. S. J., et al. (1998), Magma production and growth of the lava dome of the Soufrière Hills Volcano, Montserrat, West Indies: November 1995 to December 1997, *Geophys. Res. Lett.*, *25*, 3421–3424.

Ui, T., et al. (1999), Generation of block and ash flows during the 1990–1995 eruption of Unzen Volcano, Japan, *J. Volcanol. Geotherm. Res.*, *89*, 123–137.

Voight, B., and D. Elsworth (2000), Instability and collapse of hazardous gas-pressurised lava domes, *Geophys. Res. Lett.*, *27*, 1–4.

Voight, B., et al. (1999), Magma flow instability and cyclic activity at Soufrière Hills Volcano, Montserrat, British West Indies, *Science*, *283*(5405), 1138–1142.

Watts, R. B., et al. (2001), Growth patterns and emplacement of the andesitic lava dome at Soufrière Hills Volcano, Montserrat, in *The Eruption of Soufrière Hills Volcano, Montserrat, From 1995 to 1999*, edited by T. H. Druitt and B. P. Kokelaar, *Geol. Soc. London Mem.*, *21*, 115–152.

E. S. Calder and J. L. Palma, Department of Earth Sciences, Open University, Milton Keynes, MK7 6AA, UK. (e.s.calder@open.ac.uk)

J. A. Cortés, School of Earth and the Environment, University of Leeds, Leeds, LS2 9JT, UK.

R. Luckett, Montserrat Volcano Observatory, Montserrat, West Indies.