Journal of Volcanology and Geothermal Research 92 (1999) 31–38
Journal of volcanology and geothermal research
www.elsevier.com/locate/jvolgeores

Statistical analysis of the frequency of eruptions at Furnas Volcano, São Miguel, Azores

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Accepted 20 April 1999

Abstract

Furnas Volcano is one of three major volcanic centres on the island of São Miguel, Açores. Both Furnas and Fogo have displayed violent explosive activity since the island was first occupied in the early 15th century AD. There is concern that future volcanic activity will not only cause major economic losses, but will also result in widespread mortality, and it is for these reasons that a major programme of hazard assessment has been undertaken on Furnas. The present study is part of this programme and involves both the general statistical modelling of the record of historic eruptions and, more specifically, develops a technique for determining the rate of volcanic eruptions (λ), an important parameter in the Poisson probability model. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: frequency of eruptions; Furnas Volcano; São Miguel, Azores

1. Introduction

Furnas volcano is one of three major active volcanic centres on the island of São Miguel in the Açores. The last eruption of Furnas in 1630 AD killed ~100 people (Cole et al., 1995). Queiroz et al. (1995) have demonstrated that there was also an historic eruption ~1440 AD, during the period of the first settlement of the island by the Portuguese. The eruptions of Furnas over the last 5000 years are well documented (Booth et al., 1978, 1983; Cole et al., 1995, 1999-this issue; Guest et al., 1999-this issue) and a chronology is provided in Table 1. These eruptions range from sub-plinian/phreatomagmatic events, such as 1630 AD, to larger plinian episodes — for example Furnas C. Even the smaller eruptions, if repeated, would lead to total devastation within the caldera (Fig. 1). In an assessment of the volcanic hazard of Furnas, it is important to consider the probability of a future eruption and Moore (1990), on the basis of the number of eruptions over the last 5000 years, showed that eruptions had taken place on average once every 300 years and, with the last eruption occurring almost 400 years ago, he considered that there was at present a high probability of eruption.

Eruptive mechanisms of volcanoes are not, however, sufficiently well understood to allow determin-

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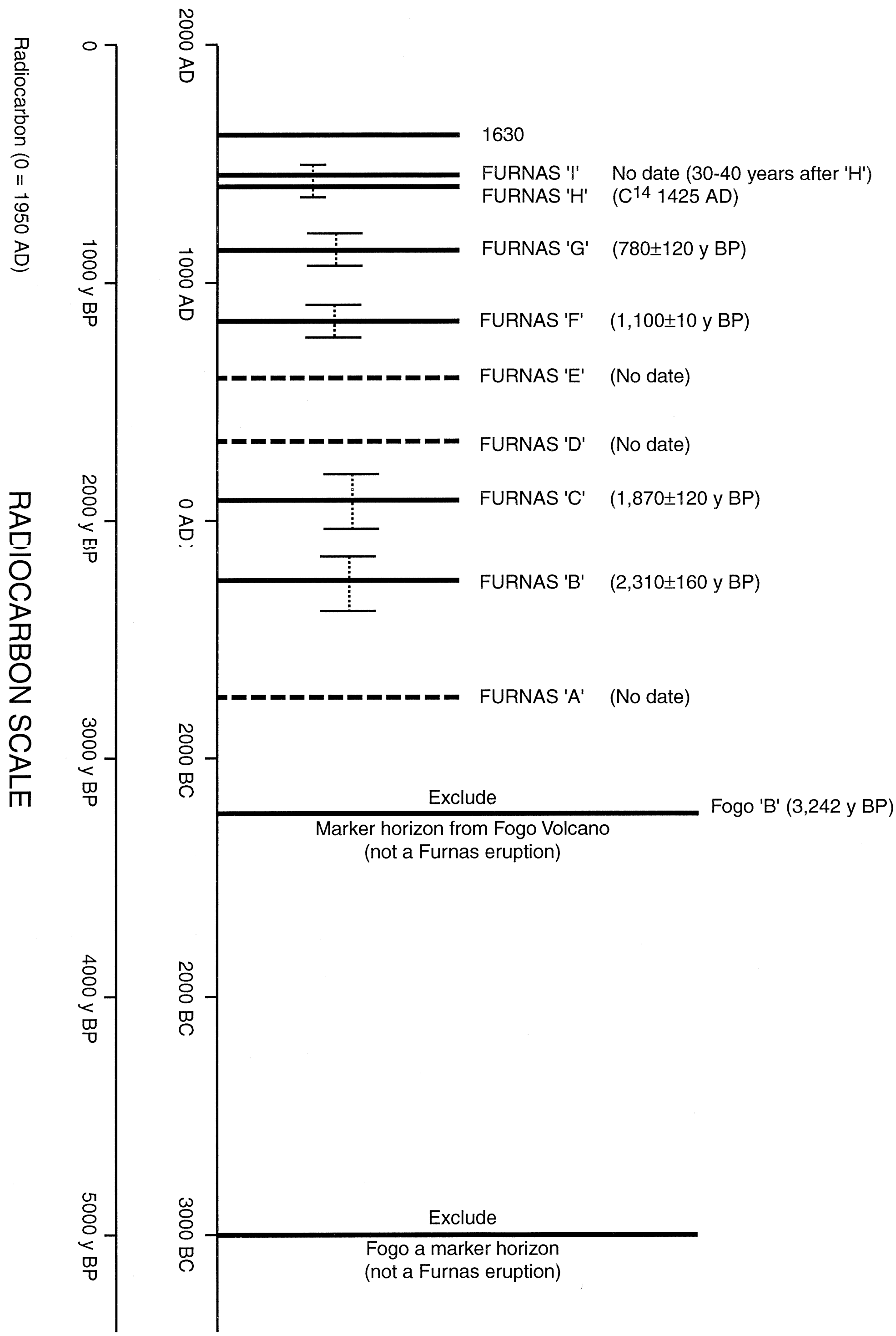
PII: S0377-0273(99)00065-7

Table 1
Eruption chronology, São Miguel, Açores (based on Queiroz et al. and other sources)



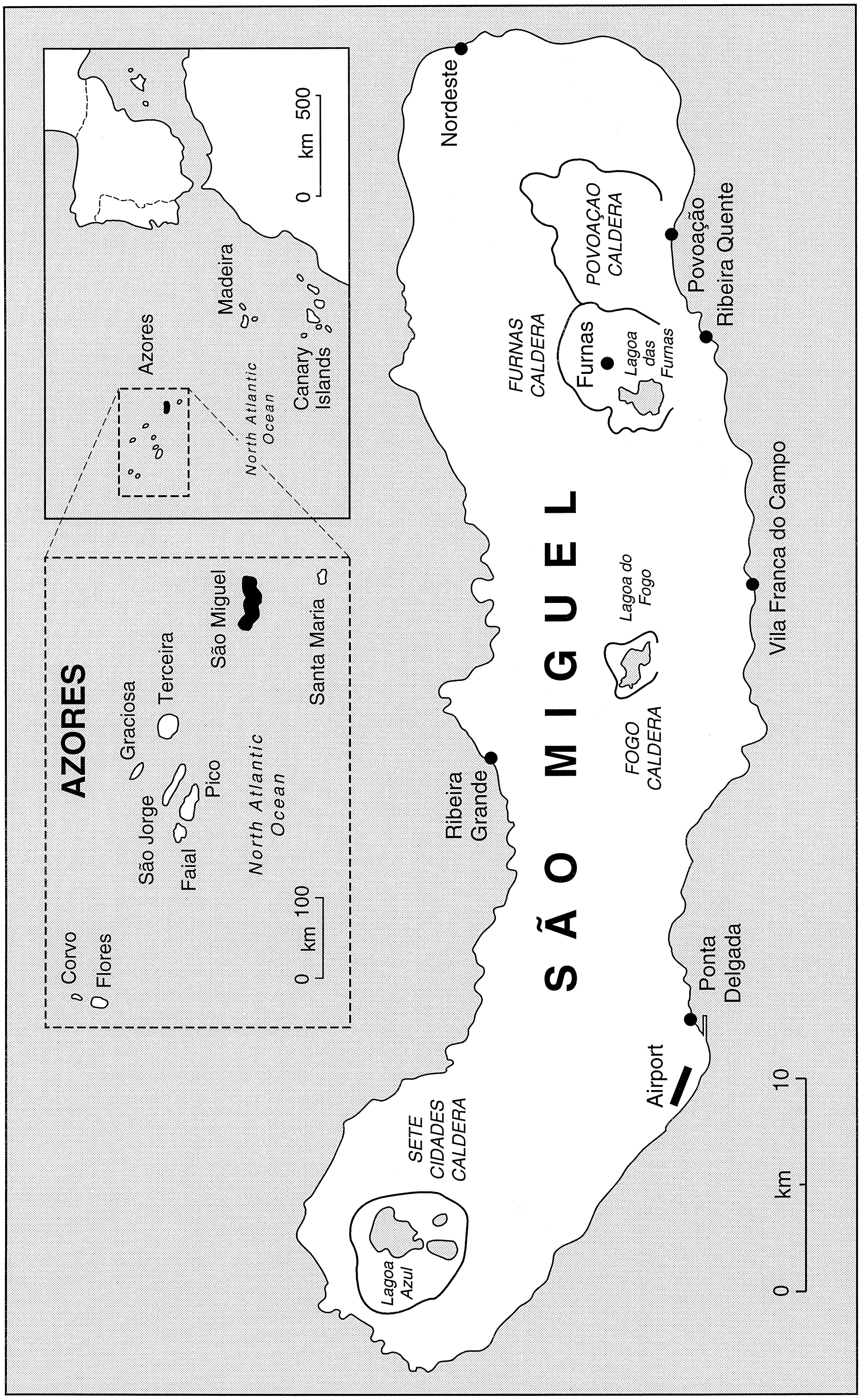


Fig. 1. Furnas Volcano: general location map (from Chester et al., 1995).

istic predictions of future activity to be made with confidence. On Furnas and many other volcanoes prediction must rely on statistical modelling using eruption records. Such models have allowed many important features of future eruptions to be predicted. These have included: the frequencies; types; spatial patterns and probabilities of eruptions, all of which have been incorporated into risk assessments (Wickman, 1966a,b,c,d,e; Klein, 1982, 1984; Newhall, 1982, 1984; Scandone, 1983; Mulargia et al., 1985; Condit et al., 1989; Cruz-Reyna, 1991; Ho et al., 1991).

In this paper we propose a statistical approach which is applicable equally to Furnas and to other volcanoes with similarly deficient data sets. In particular, we estimate the unknown parameters which are required to calculate the probability of eruptions. Taking into account the limited availability of precise data on the past eruptions of Furnas, it is our contention that estimates made from our model are meaningful, both statistically and geologically.

2. The data

Data are presented in Table 1 and dates of eruption are referred to two time axes. The first, labelled RADIOCARBON, is scaled from 5000 years BP to present, but note that radiocarbon dates have an arbitrary reference point of 0 BP = 1950 AD). The second scale is chronological, having a range from 3000 BC to 2000 AD. Both scales use intervals of 200 years. The nature of these data are such that values are estimated when they do not coincide with either of the axes (Table 1).

3. Choice of model

Application of statistical methods to volcanic eruptions began with the pioneering research of Wickman (1966a; b; c; d; e; 1976), who discussed the applicability of various Poisson models to volcanic events. His research involved the calculation of eruption recurrence rates for a number of volcanoes with different styles of activity. Wickman observed that for this class of volcano age specific eruption rates are not time dependent. Such volcanoes are 'without a memory' of previous events and, hence, are termed Simple Poissonian Volcanoes.

It should be noted that events can have:

1. A random distribution (not to be confused with the term 'random variable').
2. A regular distribution.
3. A contagious distribution.
4. A random distribution, implying that events occur 'at random'.

A Poisson distribution provides a suitable model for such behaviour and it is the hypothesis of randomness that is usually the first to be tested. A regular distribution suggests that events occur at fixed, regular intervals and a contagious distribution implies that there is a clustering of events. In this paper, therefore, a Poisson model will be fitted to the eruptions of Furnas. Before doing so it should be noted that Ho et al. (1991) has introduced the concept of a Nonhomogeneous Poisson model, that is a model in which the occurrence of an event is dependent on the elapsed time since the previous event, but Babbington and Lai (1996) argue that such a model can possess certain undesirable features.

The Poisson process is used to describe a wide variety of stochastic phenomena that share certain characteristics and in which some 'happening' — or event — takes place sporadically over time, in a manner which is commonly believed to be 'random.' If events in a *Poisson* process occur at a mean rate of λ per unit time (1 year, 105 year, etc.), then the expected number (long-run average) of occurrences in an interval of time in t units is λt (e.g., see Ho et al., 1991). Using Table 1, a volcanic eruption is defined as such an event.

The rate of occurrence of volcanic eruptions, λ is a critical parameter in the probability calculation. Various statistical methods for calculating λ are examined to show how the record of volcanism on Furnas may be used to estimate values of λ and so demonstrate the limitations involved in calculating λ.

4. Methodology

By electing to employ a Poisson model means that volcanic eruptions occur at random, according to the Poisson process. Then the number, X, of occurrences per unit time has a probability $P(X=x) = e^{-\lambda}\lambda^x/x!$, $x=0, 1, 2,\ldots$ and the time between eruptions has a negative exponential distribution whose probability density function is $f(t)=\lambda e^{-\lambda t}$, $t, \lambda>0$. Observe that λ is a parameter to be estimated from the data.

Mathematically, the assumptions for a Poisson distribution are as follows:
1. the probability of an event occurring in a small interval of time $(t, t+\delta t)$ is $\lambda t + o(\delta t)$;
2. the probability of two or more events occurring in a small interval of time $(t, t\delta t)$ is $o(\delta t)$;
3. occurrences in non-overlapping intervals are independent of one another.

The term $o(\delta t)$ denotes that terms such as δt^2 and higher powers are present, but are so small that they can be ignored. The model indicates that the length, δt of the time interval is such that at most one event can occur. Of course, λ is assumed to remain constant throughout, since this is the rate of occurrence of volcanic eruptions.

5. The analysis

Let R denote 'the number of eruptions per unit time'. Then R follows a Poisson distribution with parameter λ, where λ is the rate of volcanic eruptions. This means that the probability of $R=r$ eruptions in unit time is given by:

$$P(R=r) = e^{-\lambda}\lambda^r/r! \ \text{ for } \ r=0,1,2\ldots$$

Also, the time between eruptions follows an exponential distribution whose probability density function (pdf) is given by:

$$f(t) = \lambda e^{-\lambda t}, \ \ \lambda,t>0.$$

To obtain an estimator for λ the method of Maximum Likelihood is used. Ho et al. (1991, p. 50), argue that, ''in dealing with distributions, repeating a random experiment several times to obtain information about the unknown parameter(s) is useful. The collection of resulting observations, denoted $x_1, x_2, \ldots, x_n$ is a sample from the associated distribution. Often these observations are collected so that they are independent of each other. That is, one observation must not influence the others. If this type of independence exists, it follows that $x_1, x_2, \ldots, x_n$ are observations of a random sample of size n. The distribution from which the sample arises is the population. The observed sample values, $x_1, x_2, \ldots, x_n$, are used to determine information about the unknown population (or distribution)''. Assuming that $x_1, x_2, \ldots, x_n$ represent a random sample from a Poisson population with parameter λ, the likelihood function is:

$$L(\lambda) = \prod_{i=1}^{n} f(x_i;\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$L(\lambda) = e^{-n\lambda}\lambda^{\frac{\sum_{i=1}^{n} x_i}{\prod_{i=1}^{n} x_i!}}$$

Many statistical procedures employ values for the population parameters that 'best' explain the observed data. One meaning of 'best' is to select the parameter values that maximise the likelihood function. This technique is called maximum likelihood estimation, and the maximising parameter values are called maximum likelihood estimates, also denoted MLE, or $\hat{\lambda}$. Note that any value of λ that maximises $L(\lambda)$ will also maximises that log-likelihood, $\ln L(\lambda)$. Thus, for computational convenience, the alternate form of the maximum likelihood equation, will often be used, and the log-likelihood for a random sample from a Poisson distribution is:

$$\ln L(\lambda) = -n\lambda + \sum_{i=1}^{n} x_i \ln\lambda - \left(\prod_{i=1}^{n} x_i!\right).$$

The maximum likelihood equation is therefore:

$$\frac{d}{d\lambda}\ln L(\lambda) = -n + \sum_{i=1}^{n}\frac{x_i}{\lambda} = 0,$$

which has the solution $\hat{\lambda} = \sum_{i=1}^{n}\frac{x_i}{n} = \bar{x}$. This is indeed a maximum because the second derivative:

$$\frac{d^2}{d\lambda^2}\ln L(\lambda) = -\sum_{i=1}^{n}\frac{x_i}{\lambda^2},$$

is negative when evaluated at $\bar{x}$.

Let this estimation technique be demonstrated. Let X denote the number of volcanic eruptions for a 10^5-year period for the NTS (Nevada Test Site) region from this assumed Poisson process. Then X follows a Poisson distribution with average recurrence rate m, with $m = \lambda t = 10^5\lambda$. If we wish to estimate λ for the Quaternary using the Poisson count data for the NTS region, the successive number of eruptions from the last 16 consecutive intervals of length 105 years ($16 \times 10^5 = 1.6 \times 10^6 =$ Quaternary period) must be estimated. The number of observed eruptions per interval are denoted as x_1, x_2, ..., x_{16}. Thus, these 16 values represent a sample of size 16 from a Poisson random variable with average recurrence rate m. Estimating the mean of the Poisson variable from these count data gives:

$$\mu = \bar{x} = \sum_{i=1}^{16} \frac{x_i}{16},$$

and

$$\hat{\lambda} = \frac{\hat{\mu}}{10^5} = \sum_{i=1}^{16} \frac{x_i}{(16 \times 10^6)}$$

This shows that the estimated annual recurrence rate, $\hat{\lambda}$, is the average number of eruptions during the observation period (in years). Based on this estimation technique, $\hat{\lambda}$ can be defined as:

$$\hat{\lambda} = \frac{E}{T} \qquad (1)$$

where $E =$ total number of eruption during the observation period, and $T =$ observation period.

Note that for the estimation of λ in this model, an individual observation x_i is not required. However, the distribution of x_i's can provide information for model selection, for checking the adequacy of the model and for parameter estimation in general.

The above description indicates that Eq. (1) is the 'best' estimator arising out of the application of method of MLE. However, although we also use MLE, our method in estimating λ is different from Ho et al. (1991) and we believe it provides a better estimate of the parameter λ. We, therefore, estimate λ as follows:

First, the likelihood function $L(\lambda;t_1,t_2,\ldots,t_n)$ is set up, the natural logarithm of the equation is taken and then differentiated with respect to λ. The resulting expression is made equal to zero and the equation is solved using an appropriate method.

Turning once more to the data relevant to the Furnas research problem and taking the origin on the chronological scale (Table 1) to be 1000 BC, then:

(a) Three events occur during a period of 870 years (A + B + C).
(b) Three events take place over a 730 year period (D + E + F).
(c) One event takes place after 360 years (G).
(d) One event occurs after 265 years (H).
(e) One event occurs after 35 years (I).
(f) One event occurs after 170 years (J).
(g) No events have happened for a period of 365 years.

Given that $L(\lambda;t_1,t_2,\ldots,t_n)$ is the joint probability of the observations we may write:

$$L(\lambda;t_1,\ldots,t_n) = (1-e^{-870\lambda})^3(1-e^{-730\lambda})^3 \times \lambda e^{-360\lambda}\lambda e^{-265\lambda}\lambda e^{-35\lambda}\lambda e^{-170\lambda}e^{-365\lambda}.$$

Taking natural logarithms we have:

$$\ln L = 4\ln\lambda - 1195\lambda + 3\ln(1-e^{-870\lambda}) + 3\ln(1-e^{-730\lambda}).$$

Differentiating and rearranging terms we have:

$$\frac{d(\ln L)}{d\lambda} = \frac{4}{\lambda} - 1195 + \frac{3e^{-870\lambda}(870)}{1-e^{-870\lambda}} + \frac{3e^{-710}(710)}{1-e^{-710\lambda}} = \frac{4}{\lambda} - 1195 + \frac{5610}{e^{1870\lambda}-1} + \frac{2130}{e^{710\lambda}-1}$$

$$\frac{d(\ln L)}{d\lambda} = 0$$

Equate this to zero and solve numerically, using Program 1 shown below, to give: $\lambda = 0.00403075$.

```
10 T = 0
20 T1 = 300
30 WHILE ABS(T1 − T) > 0.0000001
40 T = T1
50 T1 = 298.75 − 1402.5/(EXP(1870 * 1/T) − 1)
− 532.5/(EXP(710 * 1/T) − 1)
60 PRINT TAB(1);1/T;TAB(20);1/T1
70 NEXT
80 END
```

Program 1

From this we can see that:

$P(T > 5) = 0.980$

$P(T > 10) = 0.960$

$P(T > 25) = 0.904$

$P(T > 50) = 0.817$

$P(T > 75) = 0.739$

$P(T > 100) = 0.668$

Thus, the probability that no eruption will occur within the next 5 years is 0.980; that is, there is a 2% chance of an eruption occurring.

The estimates above of the probability of no eruption occurring prior to a given elapsed time are based on the point estimate of λ, namely 0.00403075. We suggest that estimates based on a 95% confidence interval for λ would be more useful. It is well known that if a random variable, X, has a negative exponential distribution with parameter λ then $2n\bar{x}\lambda$ is distributed as a χ^2 random variable on $2n$ degrees of freedom. Thus, with $\bar{x} = 320$ and $n = 10$ we can set up a 95% confidence interval for λ. With 20 degrees of freedom, the lower 2.5% point is 9.591 and the upper 2.5% point is 34.170. These values lead to the 95% interval $0.00148467 < \lambda < 0.00528974$. This enables us to refine our estimates to give:

$P(T > 5) = 0.973$

$P(T > 10) = 0.947$

$P(T > 25) = 0.868$

$P(T > 50) = 0.736$

$P(T > 75) = 0.603$

$P(T > 100) = 0.471$

Thus, the revised probability that no eruption occurring within the next 5 years is 0.973; that is, there is a 3% chance of an eruption occurring.

6. Conclusions

The problem discussed in this paper involves the derivation of a technique for estimating the rate of eruptions by means of (λ), an important parameter in the Poisson probability model. The method is based on the number of eruptions occurring over a defined period of observation. A recently published analysis of global volcanic activity argues (Ho et al., 1991) that the number of eruptions occurring per unit time, also follows a Poisson distribution. By using the method of maximum likelihood, an estimated value for λ has been derived. Further, a 95% interval estimate for λ has been estimated and this may be used to provide interval estimates of probabilities of eruption over a range of times from 5 to 100 years.

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